

FORBIDDEN PAIRS AND (k, m) -PANCYCLICITY

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Abstract

A graph G on n vertices is said to be (k, m) -pancyclic if every set of k vertices in G is contained in a cycle of length r for each $r \in \{m, m+1, \dots, n\}$. This property, which generalizes the notion of a vertex pancyclic graph, was defined by Faudree, Gould, Jacobson, and Lesniak in 2004. The notion of (k, m) -pancyclicity provides one way to measure the prevalence of cycles in a graph. We consider pairs of subgraphs that, when forbidden, guarantee hamiltonicity for 2-connected graphs on $n \geq 10$ vertices. There are exactly ten such pairs. For each integer $k \geq 1$ and each of eight such subgraph pairs $\{R, S\}$, we determine the smallest value m such that any 2-connected $\{R, S\}$ -free graph on $n \geq 10$ vertices is guaranteed to be (k, m) -pancyclic. Examples are provided that show the given values are best possible. Each such example we provide represents an infinite family of graphs.

Keywords: hamiltonian, pancyclic, forbidden subgraph, cycle, claw-free.

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