

THE THICKNESS OF AMALGAMATIONS AND CARTESIAN PRODUCT OF GRAPHS

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Abstract

The thickness of a graph is the minimum number of planar spanning subgraphs into which the graph can be decomposed. It is a measurement of the closeness to the planarity of a graph, and it also has important applications to VLSI design, but it has been known for only few graphs. We obtain the thickness of vertex-amalgamation and bar-amalgamation of graphs, the lower and upper bounds for the thickness of edge-amalgamation and 2-vertex-amalgamation of graphs, respectively. We also study the thickness of Cartesian product of graphs, and by using operations on graphs, we derive the thickness of the Cartesian product $K_n \square P_m$ for most values of m and n .

Keywords: thickness, amalgamation, Cartesian product, genus.

2010 Mathematics Subject Classification: 05C10.

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Received 29 June 2015

Revised 18 May 2016

Accepted 20 May 2016