Discussiones Mathematicae
Graph Theory 37 (2017) 623–632
doi:10.7151/dmgt.1941

Full PDF
DMGT Page

ON THE MAXIMUM AND MINIMUM SIZES OF A GRAPH WITH GIVEN k-CONNECTIVITY

Yuefang Sun

Department of Mathematics
Shaoxing University
Zhejiang 312000, P.R. China

e-mail: yfsun2013@gmail.com

Abstract

The concept of k-connectivity $\kappa_k(G)$, introduced by Chartrand in 1984, is a generalization of the cut-version of the classical connectivity. For an integer $k \geq 2$, the k-connectivity of a connected graph G with order $n \geq k$ is the smallest number of vertices whose removal from G produces a graph with at least k components or a graph with fewer than k vertices. In this paper, we get a sharp upper bound for the size of G with $\kappa_k(G) = t$, where $1 \leq t \leq n - k$ and $k \geq 3$; moreover, the unique extremal graph is given. Based on this result, we get the exact values for the maximum size, denoted by $g(n, k, t)$, of a connected graph G with order n and $\kappa_k(G) = t$. We also compute the exact values and bounds for another parameter $f(n, k, t)$ which is defined as the minimum size of a connected graph G with order n and $\kappa_k(G) = t$, where $1 \leq t \leq n - k$ and $k \geq 3$.

Keywords: k-connectivity, generalized connectivity, connectivity.

2010 Mathematics Subject Classification: 05C35, 05C40.

References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, Berlin, 2008).

[2] G. Chartrand, S.F. Kappor, L. Lesniak and D.R. Lick, *Generalized connectivity in graphs*, Bull. Bombay Math. Colloq. **2** (1984) 1–6.

[3] G. Chartrand, F. Okamoto and P. Zhang, *Rainbow trees in graphs and generalized connectivity*, Networks **55** (2010) 360–367.

[4] D.P. Day, O.R. Oellermann and H.C. Swart, *The ℓ-connectivity function of trees and complete multipartite graphs*, J. Combin. Math. Combin. Comput. **10** (1991) 183–192.

[5] R. Gu, X. Li and Y. Shi, *The generalized* 3*-connectivity of random graphs*, Acta Math. Sinica **57** (2014) 321–330, in Chinese.

[6] M. Hager, *Pendant tree-connectivity*, J. Combin. Theory Ser. B **38** (1985) 179–189. doi:10.1016/0095-8956(85)90083-8

[7] H. Li, X. Li, Y. Mao and Y. Sun, *Note on the generalized connectivity*, Ars Combin. **114** (2014) 193–202.

[8] H. Li, X. Li and Y. Sun, *The generalized* 3*-connectivity of Cartesian product graphs*, Discrete Math. Theor. Comput. Sci. **14** (2012) 43–54. doi:10.1016/j.commatsci.2011.09.003

[9] X. Li and Y. Mao, *A survey on the generalized connectivity of graphs*. arXiv:1207.1838[math.CO]

[10] X. Li and Y. Mao, *On extremal graphs with at most* ℓ *internally disjoint Steiner trees connecting any* $n-1$ *vertices*, Graphs Combin. **31** (2015) 2231–2259. doi:10.1007/s00373-014-1500-7

[11] X. Li and Y. Mao, *The generalized* 3*-connectivity of lexicographic product graphs*, Discrete Math. Theor. Comput. Sci. **16** (2014) 339–354. doi:10.1007/978-3-319-12691-3_31

[12] X. Li, Y. Mao and Y. Sun, *On the generalized* (*edge*)*-connectivity of graphs*, Australas. J. Combin. **58** (2014) 304–319.

[13] O.R. Oellermann, *On the* ℓ*-connectivity of a graph*, Graphs Combin. **3** (1987) 285–299. doi:10.1007/BF01788551

[14] O.R. Oellermann, *A note on the* ℓ*-connectivity function of a graph*, Congr. Numer. **60** (1987) 181–188.

[15] Y. Sun, *Generalized* 3*-*(*edge*)*-connectivity for undirected double-loop networks*, J. Discrete Math. Sci. Cryptogr. **17** (2014) 19–28. doi:10.1080/09720529.2013.867672

[16] Y. Sun, *Generalized* 3*-edge-connectivity of Cartesian product graphs*, Czechoslovak Math. J. **65** (2015) 107–117. doi:10.1007/s10587-015-0162-9

[17] Y. Sun, *Generalized* 3*-connectivity and* 3*-edge-connectivity for the Cartesian products of some graph classes*, J. Combin. Math. Combin. Comput. **94** (2015) 215–225.

[18] Y. Sun, *Maximum generalized local connectivities of cubic Cayley graphs on Abelian groups*, J. Combin. Math. Combin. Comput. **94** (2015) 227–236.

[19] Y. Sun, *Sharp upper bounds for generalized edge-connectivity of product graphs*, Discuss. Math. Graph Theory **36** (2016) 833–843. doi:10.7151/dmgt.1924

[20] Y. Sun, *A sharp lower bound for the generalized* 3*-edge-connectivity of strong product graphs*, Discuss. Math. Graph Theory, accepted.

[21] Y. Sun, F. Li and Z. Jin, *On two generalized connectivities of graphs*, Discuss. Math. Graph Theory, accepted.

[22] Y. Sun and X. Li, *On the difference of two generalized connectivities of a graph*, J. Comb. Optim. (2015), in press.
doi:10.1007/s10878-015-9956-9

[23] Y. Sun and S. Zhou, *Tree connectivities of Cayley graphs on Abelian groups with small degrees*, Bull. Malays. Math. Sci. Soc. **39** (2016) 1673–1685.
doi:10.1007/s40840-015-0147-8

Received 19 January 2016
Revised 27 May 2016
Accepted 1 June 2016