

A SUFFICIENT CONDITION FOR GRAPHS TO BE SUPER k -RESTRICTED EDGE CONNECTED¹

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Abstract

For a subset S of edges in a connected graph G , S is a k -restricted edge cut if $G - S$ is disconnected and every component of $G - S$ has at least k vertices. The k -restricted edge connectivity of G , denoted by $\lambda_k(G)$, is defined as the cardinality of a minimum k -restricted edge cut. Let $\xi_k(G) = \min\{|[X, \bar{X}]| : |X| = k, G[X] \text{ is connected}\}$, where $\bar{X} = V(G) \setminus X$. A graph G is super k -restricted edge connected if every minimum k -restricted edge cut of G isolates a component of order exactly k . Let k be a positive integer and let G be a graph of order $\nu \geq 2k$. In this paper, we show that if $|N(u) \cap N(v)| \geq k + 1$ for all pairs u, v of nonadjacent vertices and $\xi_k(G) \leq \lfloor \frac{\nu}{2} \rfloor + k$, then G is super k -restricted edge connected.

Keywords: graph, neighborhood, k -restricted edge connectivity, super k -restricted edge connected graph.

2010 Mathematics Subject Classification: 05C40.

¹This work is supported by the National Science Foundation of China (61370001).

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Received 13 August 2015

Revised 11 May 2016

Accepted 11 May 2016