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A SUFFICIENT CONDITION FOR GRAPHS TO BE SUPER k-RESTRICTED EDGE CONNECTED[1]

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Abstract

For a subset S of edges in a connected graph G, S is a k-restricted edge cut if $G-S$ is disconnected and every component of $G-S$ has at least k vertices. The k-restricted edge connectivity of G, denoted by $\lambda_k(G)$, is defined as the cardinality of a minimum k-restricted edge cut. Let $\xi_k(G) = \min\{|[X,\bar{X}]| : |X| = k,\ G[X] \text{ is connected}\}$, where $\bar{X} = V(G)\backslash X$. A graph G is super k-restricted edge connected if every minimum k-restricted edge cut of G isolates a component of order exactly k. Let k be a positive integer and let G be a graph of order $\nu \geq 2k$. In this paper, we show that if $|N(u) \cap N(v)| \geq k+1$ for all pairs u, v of nonadjacent vertices and $\xi_k(G) \leq \lfloor \frac{\nu}{2} \rfloor + k$, then G is super k-restricted edge connected.

Keywords: graph, neighborhood, k-restricted edge connectivity, super k-restricted edge connected graph.

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