

A TRIPLE OF HEAVY SUBGRAPHS ENSURING PANCYCLICITY OF 2-CONNECTED GRAPHS

WOJCIECH WIDĘŁ

Faculty of Applied Mathematics
Department of Discrete Mathematics
AGH University of Science and Technology
al. Mickiewicza 30, 30–059 Krakow, Poland

e-mail: widel@agh.edu.pl

Abstract

A graph G on n vertices is said to be pancylic if it contains cycles of all lengths k for $k \in \{3, \dots, n\}$. A vertex $v \in V(G)$ is called super-heavy if the number of its neighbours in G is at least $(n+1)/2$. For a given graph H we say that G is H - f_1 -heavy if for every induced subgraph K of G isomorphic to H and every two vertices $u, v \in V(K)$, $d_K(u, v) = 2$ implies that at least one of them is super-heavy. For a family of graphs \mathcal{H} we say that G is \mathcal{H} - f_1 -heavy, if G is H - f_1 -heavy for every graph $H \in \mathcal{H}$.

Let D denote the deer, a graph consisting of a triangle with two disjoint paths P_3 adjoined to two of its vertices. In this paper we prove that every 2-connected $\{K_{1,3}, P_7, D\}$ - f_1 -heavy graph on $n \geq 14$ vertices is pancylic. This result extends the previous work by Faudree, Ryjáček and Schiermeyer.

Keywords: cycle, Fan-type heavy subgraph, Hamilton cycle, pancylicity.

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