

A TRIPLE OF HEAVY SUBGRAPHS ENSURING PANCYCLICITY OF 2-CONNECTED GRAPHS

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Abstract

A graph G on n vertices is said to be pancyclic if it contains cycles of all lengths k for $k \in \{3, \dots, n\}$. A vertex $v \in V(G)$ is called super-heavy if the number of its neighbours in G is at least $(n+1)/2$. For a given graph H we say that G is H - f_1 -heavy if for every induced subgraph K of G isomorphic to H and every two vertices $u, v \in V(K)$, $d_K(u, v) = 2$ implies that at least one of them is super-heavy. For a family of graphs $\mathcal{H}$ we say that G is $\mathcal{H}$ - f_1 -heavy, if G is H - f_1 -heavy for every graph $H \in \mathcal{H}$.

Let D denote the deer, a graph consisting of a triangle with two disjoint paths P_3 adjoined to two of its vertices. In this paper we prove that every 2-connected $\{K_{1,3}, P_7, D\}$ - f_1 -heavy graph on $n \geq 14$ vertices is pancyclic. This result extends the previous work by Faudree, Ryjáček and Schiermeyer.

Keywords: cycle, Fan-type heavy subgraph, Hamilton cycle, pancyclicity.

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REFERENCES

- [1] P. Bedrossian, Forbidden Subgraph and Minimum Degree Conditions for Hamiltonicity (PhD Thesis, Memphis State University, USA, 1991).
- [2] P. Bedrossian, G. Chen and R.H. Schelp, *A generalization of Fan's condition for Hamiltonicity, pancyclicity and Hamiltonian connectedness*, Discrete Math. **115** (1993) 39–59.
doi:10.1016/0012-365X(93)90476-A
- [3] A. Benhocine and A.P. Wojda, *The Geng-Hua Fan conditions for pancyclic or Hamilton-connected graphs*, J. Combin. Theory Ser. B **58** (1987) 167–180.
doi:10.1016/0095-8956(87)90038-4

- [4] J.A. Bondy, *Pancyclic graphs I*, J. Combin. Theory Ser. B **11** (1971) 80–84.
doi:10.1016/0095-8956(71)90016-5
- [5] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications* (Macmillan London and Elsevier, 1976).
doi:10.1007/978-1-349-03521-2
- [6] G. Chen, B. Wei and X. Zhang, *Degree light-free graphs and Hamiltonian cycles*, Graphs Combin. **17** (2001) 409–434.
doi:10.1007/s003730170018
- [7] R. Faudree, Z. Ryjáček and I. Schiermeyer, *Forbidden subgraphs and cycle extendability*, J. Combin. Math. Combin. Comput. **19** (1995) 109–128.
- [8] M. Ferrara, M.S. Jacobson and A. Harris, *Cycle lengths in Hamiltonian graphs with a pair of vertices having large degree sum*, Graphs Combin. **26** (2010) 215–223.
doi:10.1007/s00373-010-0915-z
- [9] B. Ning, *Fan-type degree condition restricted to triples of induced subgraphs ensuring Hamiltonicity*, Inform. Process. Lett. **113** (2013) 823–826.
doi:10.1016/j.ipl.2013.07.014
- [10] B. Ning, *Pairs of Fan-type heavy subgraphs for pancyclicity of 2-connected graphs*, Australas. J. Combin. **58** (2014) 127–136.
- [11] E.F. Schmeichel and S.L. Hakimi, *A cycle structure theorem for Hamiltonian graphs*, J. Combin. Theory Ser. B **45** (1988) 99–107.
doi:10.1016/0095-8956(88)90058-5
- [12] W. Wideł, *A Fan-type heavy pair of subgraphs for pancyclicity of 2-connected graphs*, Discuss. Math. Graph Theory **36** (2016) 173–184.
doi:10.7151/dmgt.1840

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