CHARACTERIZATION RESULTS FOR THE L(2, 1, 1)-LABELING PROBLEM ON TREES

XIAOLING ZHANG

College of Mathematics and Computer Science
Quanzhou Normal University
Quanzhou 362000, Fujian, P.R. China

E-mail: xml000999@163.com

AND

KECAI DENG

School of Mathematical Science
Huaqiao University
Quanzhou 362000, Fujian, P.R. China

E-mail: kecaideng@126.com

Abstract

An L(2, 1, 1)-labeling of a graph G is an assignment of non-negative integers (labels) to the vertices of G such that adjacent vertices receive labels with difference at least 2, and vertices at distance 2 or 3 receive distinct labels. The span of such a labelling is the difference between the maximum and minimum labels used, and the minimum span over all L(2, 1, 1)-labelings of G is called the L(2, 1, 1)-labeling number of G, denoted by \( \lambda_{2,1,1}(G) \). It was shown by King, Ras and Zhou in [The L(h, 1, 1)-labelling problem for trees, European J. Combin. 31 (2010) 1295–1306] that every tree T has \( \Delta_2(T) - 1 \leq \lambda_{2,1,1}(T) \leq \Delta_2(T) \), where \( \Delta_2(T) = \max_{uv \in E(T)}(d(u) + d(v)) \). And they conjectured that almost all trees have the L(2, 1, 1)-labeling number attain the lower bound. This paper provides some sufficient conditions for \( \lambda_{2,1,1}(T) = \Delta_2(T) \). Furthermore, we show that the sufficient conditions we provide are also necessary for trees with diameter at most 6.

Keywords: L(2, 1, 1)-labeling, tree, diameter.

2010 Mathematics Subject Classification: 05C15.

References


Received 13 September 2015
Revised 27 May 2016
Accepted 27 May 2016