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CHARACTERIZATION RESULTS FOR THE $L(2,1,1)$-LABELING PROBLEM ON TREES

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Abstract

An $L(2,1,1)$-labeling of a graph G is an assignment of non-negative integers (labels) to the vertices of G such that adjacent vertices receive labels with difference at least 2, and vertices at distance 2 or 3 receive distinct labels. The span of such a labelling is the difference between the maximum and minimum labels used, and the minimum span over all $L(2,1,1)$-labelings of G is called the $L(2,1,1)$-labeling number of G, denoted by $\lambda_{2,1,1}(G)$. It was shown by King, Ras and Zhou in [*The $L(h,1,1)$-labelling problem for trees*, European J. Combin. **31** (2010) 1295–1306] that every tree T has $\Delta_2(T)-1 \le \lambda_{2,1,1}(T) \le \Delta_2(T)$, where $\Delta_2(T) = \max_{uv\in E(T)}(d(u)+d(v))$. And they conjectured that almost all trees have the $L(2,1,1)$-labeling number attain the lower bound. This paper provides some sufficient conditions for $\lambda_{2,1,1}(T) = \Delta_2(T)$. Furthermore, we show that the sufficient conditions we provide are also necessary for trees with diameter at most 6.

Keywords: $L(2,1,1)$-labeling, tree, diameter.

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References

[1] T. Calamoneri, *The $L(h,k)$-labelling problem: an updated survey and annotated bibliography*, Comput. J. **54** (2011) 1344–1371.
doi:10.1093/comjnl/bxr037

[2] T. Calamoneri, *The $L(h,k)$-labelling problem: an updated survey and annotated bibliography*, Comput. J. **54** (2011) 1344–1371.
doi:10.1093/comjnl/bxr037

[3] G.J. Chang and D. Kuo, *The $L(2,1)$-labeling problem on graphs*, SIAM J. Discrete Math. **9** (1996) 309–316.
doi:10.1137/S0895480193245339

[4] M. Chia, D. Kuo, H. Liao, C. Yang and R.K. Yeh, *$L(3,2,1)$-labeling of graphs*, Taiwanese J. Math. **15** (2011) 2439–2457.

[5] J. Fiala, P.A. Golovach, J. Kratochvíl, B. Lidický and D. Paulusma, *Distance three labelings of trees*, Discrete Appl. Math. **160** (2012) 764–779.
doi:10.1016/j.dam.2011.02.004

[6] J.P. Georges and D.W. Mauro, *Labeling trees with a condition at distance two*, Discrete Math. **269** (2003) 127–148.
doi:10.1016/S0012-365X(02)00750-1

[7] P.A. Golovach, B. Lidický and D. Paulusma, *$L(2,1,1)$-labeling is NP-complete for trees*, in: Theory and Applications of Models of Computation, Lecture Notes in Comput. Sci. **6108** (2010) 211–221.
doi:10.1007/978-3-642-13562-0_20

[8] J.R. Griggs and R.K. Yeh, *Labelling graphs with a condition at distance* 2, SIAM J. Discrete Math. **5** (1992) 586–595.
doi:10.1137/0405048

[9] W.K. Hale, *Frequency assignment: Theory and applications*, Proc. IEEE **68** (1980) 1497–1514.
doi:10.1109/PROC.1980.11899

[10] T. Hasunuma, T. Ishii, H. Ono and Y. Uno, *A linear time algorithm for $L(2,1)$-labeling of trees*, Algorithmica **66** (2013) 654–681.
doi:10.1007/s00453-012-9657-z

[11] B. Kim, B. Song and W. Hwang, *Distance three labelings for direct products of three complete graphs*, Taiwanese J. Math. **17** (2013) 207–219.
doi:10.11650/tjm.17.2013.1909

[12] D. King, C.J. Ras and S. Zhou, *The $L(h,1,1)$-labelling problem for trees*, European J. Combin. **31** (2010) 1295–1306.
doi:10.1016/j.ejc.2009.11.006

[13] W.-F. Wang, *The $L(2,1)$-labelling of trees*, Discrete Appl. Math. **154** (2006) 598–603.
doi:10.1016/j.dam.2005.09.007

[14] R.K. Yeh, *A survey on labeling graphs with a condition at distance two*, Discrete Math. **306** (2006) 1217–1231.
doi:10.1016/j.disc.2005.11.029

[15] S. Zhou, *A distance-labelling problem for hypercubes*, Discrete Appl. Math. **156** (2008) 2846–2854.
doi:10.1016/j.dam.2007.11.018

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