

## CHARACTERIZATION RESULTS FOR THE $L(2, 1, 1)$ -LABELING PROBLEM ON TREES

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### Abstract

An  $L(2, 1, 1)$ -labeling of a graph  $G$  is an assignment of non-negative integers (labels) to the vertices of  $G$  such that adjacent vertices receive labels with difference at least 2, and vertices at distance 2 or 3 receive distinct labels. The span of such a labelling is the difference between the maximum and minimum labels used, and the minimum span over all  $L(2, 1, 1)$ -labelings of  $G$  is called the  $L(2, 1, 1)$ -labeling number of  $G$ , denoted by  $\lambda_{2,1,1}(G)$ . It was shown by King, Ras and Zhou in [*The  $L(h, 1, 1)$ -labelling problem for trees*, European J. Combin. **31** (2010) 1295–1306] that every tree  $T$  has  $\Delta_2(T) - 1 \leq \lambda_{2,1,1}(T) \leq \Delta_2(T)$ , where  $\Delta_2(T) = \max_{uv \in E(T)} (d(u) + d(v))$ . And they conjectured that almost all trees have the  $L(2, 1, 1)$ -labeling number attain the lower bound. This paper provides some sufficient conditions for  $\lambda_{2,1,1}(T) = \Delta_2(T)$ . Furthermore, we show that the sufficient conditions we provide are also necessary for trees with diameter at most 6.

**Keywords:**  $L(2, 1, 1)$ -labeling, tree, diameter.

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