

THE DICHROMATIC NUMBER OF INFINITE FAMILIES OF CIRCULANT TOURNAMENTS

NAHID JAVIER AND BERNARDO LLANO

*Departamento de Matemáticas
Universidad Autónoma Metropolitana Iztapalapa
San Rafael Atlixco 186, Colonia Vicentina
09340, México, D.F., Mexico*

e-mail: {nahid,llano}@xanum.uam.mx

Abstract

The *dichromatic number* $dc(D)$ of a digraph D is defined to be the minimum number of colors such that the vertices of D can be colored in such a way that every chromatic class induces an acyclic subdigraph in D . The *cyclic* circulant tournament is denoted by $T = \vec{C}_{2n+1}(1, 2, \dots, n)$, where $V(T) = \mathbb{Z}_{2n+1}$ and for every jump $j \in \{1, 2, \dots, n\}$ there exist the arcs $(a, a + j)$ for every $a \in \mathbb{Z}_{2n+1}$. Consider the circulant tournament $\vec{C}_{2n+1}\langle k \rangle$ obtained from the cyclic tournament by reversing one of its jumps, that is, $\vec{C}_{2n+1}\langle k \rangle$ has the same arc set as $\vec{C}_{2n+1}(1, 2, \dots, n)$ except for $j = k$ in which case, the arcs are $(a, a - k)$ for every $a \in \mathbb{Z}_{2n+1}$. In this paper, we prove that $dc(\vec{C}_{2n+1}\langle k \rangle) \in \{2, 3, 4\}$ for every $k \in \{1, 2, \dots, n\}$. Moreover, we classify which circulant tournaments $\vec{C}_{2n+1}\langle k \rangle$ are vertex-critical r -dichromatic for every $k \in \{1, 2, \dots, n\}$ and $r \in \{2, 3, 4\}$. Some previous results by Neumann-Lara are generalized.

Keywords: tournament, dichromatic number, vertex-critical r -dichromatic tournament.

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