

## SHARP UPPER BOUNDS FOR GENERALIZED EDGE-CONNECTIVITY OF PRODUCT GRAPHS

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### Abstract

The generalized  $k$ -connectivity  $\kappa_k(G)$  of a graph  $G$  was introduced by Hager in 1985. As a natural counterpart of this concept, Li *et al.* in 2011 introduced the concept of generalized  $k$ -edge-connectivity which is defined as  $\lambda_k(G) = \min\{\lambda(S) : S \subseteq V(G) \text{ and } |S| = k\}$ , where  $\lambda(S)$  denote the maximum number  $\ell$  of pairwise edge-disjoint trees  $T_1, T_2, \dots, T_\ell$  in  $G$  such that  $S \subseteq V(T_i)$  for  $1 \leq i \leq \ell$ . In this paper, we study the generalized edge-connectivity of product graphs and obtain sharp upper bounds for the generalized 3-edge-connectivity of Cartesian product graphs and strong product graphs. Among our results, some special cases are also discussed.

**Keywords:** generalized edge-connectivity, Cartesian product, strong product, lexicographic product.

**2010 Mathematics Subject Classification:** 05C40, 05C76.

### REFERENCES

- [1] J.A. Bondy and U.S.R. Murty, Graph Theory, GTM 244 (Springer, Berlin, 2008).
- [2] G. Chartrand, S.F. Kappor, L. Lesniak and D.R. Lick, *Generalized connectivity in graphs*, Bull. Bombay Math. Colloq. **2** (1984) 1–6.
- [3] G. Chartrand, F. Okamoto and P. Zhang, *Rainbow trees in graphs and generalized connectivity*, Networks **55** (2010) 360–367.  
doi:10.1002/net.20339
- [4] D.P. Day, O.R. Oellermann and H.C. Swart, *The  $\ell$ -connectivity function of trees and complete multipartite graphs*, J. Combin. Math. Combin. Comput. **10** (1991) 183–192.

- [5] M. Hager, *Pendant tree-connectivity*, J. Combin. Theory Ser. B **38** (1985) 179–189.  
doi:10.1016/0095-8956(85)90083-8
- [6] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs*, Second Edition (CRC Press, 2011).
- [7] W. Imrich and S. Klavžar, *Product Graphs—Structure and Recognition* (Wiley, New York, 2000).
- [8] W. Imrich, S. Klavžar and D.F. Rall, *Topics in Graph Theory: Graphs and Their Cartesian Product* (Taylor & Francis, 2008).
- [9] H. Li, X. Li, Y. Mao and Y. Sun, *Note on the generalized connectivity*, Ars Combin. **114** (2014) 193–202.
- [10] H. Li, X. Li and Y. Sun, *The generalized 3-connectivity of Cartesian product graphs*, Discrete Math. Theor. Comput. Sci. **14** (2012) 43–54.  
doi:10.1016/j.commatsci.2011.09.003
- [11] X. Li and Y. Mao, *A survey on the generalized connectivity of graphs*.  
arXiv:1207.1838[math.CO]
- [12] X. Li and Y. Mao, *On extremal graphs with at most  $\ell$  internally disjoint Steiner trees connecting any  $n - 1$  vertices*, Graphs Combin. **31** (2015) 2231–2259.  
doi:10.1007/s00373-014-1500-7
- [13] X. Li and Y. Mao, *The generalized 3-connectivity of lexicographic product graphs*, Discrete Math. Theor. Comput. Sci. **16** (2014) 339–354.  
doi:10.1007/978-3-319-12691-3.31
- [14] X. Li and Y. Mao, *Nordhaus-Gaddum-type results for the generalized edge-connectivity of graphs*, Discrete Appl. Math. **185** (2015) 102–112.  
doi:10.1016/j.dam.2014.12.009
- [15] X. Li, Y. Mao and Y. Sun, *On the generalized (edge-)connectivity of graphs*, Australas. J. Combin. **58** (2014) 304–319.
- [16] X. Li, J. Yue and Y. Zhao, *The generalized 3-edge-connectivity of lexicographic product graphs*, in: Proc. COCOA 2014, Lecture Notes in Comput. Sci. **8881** (2014) 412–425.  
doi:10.1007/978-3-319-12691-3.31
- [17] B. Liouville, *Sur la connectivité des produits de graphes*, C.R. Acad. Sci. Paris Sér. A–B **286** (1978) A363–A365.
- [18] O.R. Oellermann, *On the  $\ell$ -connectivity of a graph*, Graphs Combin. **3** (1987) 285–299.  
doi:10.1007/BF01788551
- [19] O.R. Oellermann, *A note on the  $\ell$ -connectivity function of a graph*, Congr. Numer. **60** (1987) 181–188.
- [20] G. Sabidussi, *Graphs with given group and given graph theoretical properties*, Canadian J. Math. **9** (1957) 515–525.

- [21] S. Špacapan, *Connectivity of Cartesian products of graphs*, Appl. Math. Lett. **21** (2008) 682–685.  
doi:10.1016/j.aml.2007.06.010
- [22] Y. Sun, *Generalized 3-(edge)-connectivity for undirected double-loop networks*, J. Discrete Math. Sci. Cryptogr. **17** (2014) 19–28.  
doi:10.1080/09720529.2013.867672
- [23] Y. Sun, *Generalized 3-edge-connectivity of Cartesian product graphs*, Czechoslovak Math. J. **65** (2015) 107–117.  
doi:10.1007/s10587-015-0162-9
- [24] Y. Sun, *Generalized 3-connectivity and 3-edge-connectivity for the Cartesian products of some graph classes*, J. Combin. Math. Combin. Comput. **94** (2015) 215–225.
- [25] Y. Sun, *Maximum generalized local connectivities of cubic Cayley graphs on Abelian groups*, J. Combin. Math. Combin. Comput. **94** (2015) 227–236.
- [26] Y. Sun, F. Li and Z. Jin, *On two generalized connectivities of graphs*, submitted.
- [27] Y. Sun, *A sharp lower bound for the generalized 3-edge-connectivity of strong product graphs*, accepted for publication in Discuss. Math. Graph Theory.
- [28] Y. Sun, *On the maximum and minimum sizes of a graph with given  $k$ -connectivity*, accepted for publication in Discuss. Math. Graph Theory.
- [29] Y. Sun and X. Li, *On the difference of two generalized connectivities of a graph*, J. Comb. Optim. (2015), in press.  
doi:10.1007/s10878-015-9956-9
- [30] Y. Sun and S. Zhou, *Tree connectivities of Cayley graphs on Abelian groups with small degrees*, Bull. Malays. Math. Sci. Soc. **39** (2016) 1673–1685.  
doi:10.1007/s40840-015-0147-8
- [31] J. Xu and C. Yang, *Connectivity of Cartesian product graphs*, Discrete Math. **306** (2006) 159–165.  
doi:10.1016/j.disc.2005.11.010
- [32] C. Yang and J. Xu, *Connectivity and edge-connectivity of strong product graphs*, J. Univ. Sci. Technol. China **38** (2008) 449–455.

Received 24 September 2015

Revised 16 December 2015

Accepted 16 December 2015