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SHARP UPPER BOUNDS FOR GENERALIZED EDGE-CONNECTIVITY OF PRODUCT GRAPHS

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Abstract

The generalized k-connectivity $\kappa_k(G)$ of a graph G was introduced by Hager in 1985. As a natural counterpart of this concept, Li *et al.* in 2011 introduced the concept of generalized k-edge-connectivity which is defined as $\lambda_k(G) = \min\{\lambda(S) : S \subseteq V(G) \text{ and } |S| = k\}$, where $\lambda(S)$ denote the maximum number ℓ of pairwise edge-disjoint trees $T_1, T_2, \ldots, T_\ell$ in G such that $S \subseteq V(T_i)$ for $1 \leq i \leq \ell$. In this paper, we study the generalized edge-connectivity of product graphs and obtain sharp upper bounds for the generalized 3-edge-connectivity of Cartesian product graphs and strong product graphs. Among our results, some special cases are also discussed.

Keywords: generalized edge-connectivity, Cartesian product, strong product, lexicographic product.

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