

SOME RESULTS ON 4-TRANSITIVE DIGRAPHS

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Abstract

Let D be a digraph with set of vertices V and set of arcs A . We say that D is k -transitive if for every pair of vertices $u, v \in V$, the existence of a uv -path of length k in D implies that $(u, v) \in A$. A 2-transitive digraph is a transitive digraph in the usual sense.

A subset N of V is k -independent if for every pair of vertices $u, v \in N$, we have $d(u, v), d(v, u) \geq k$; it is l -absorbent if for every $u \in V \setminus N$ there exists $v \in N$ such that $d(u, v) \leq l$. A k -kernel of D is a k -independent and $(k - 1)$ -absorbent subset of V . The problem of determining whether a digraph has a k -kernel is known to be $\mathcal{NP}$ -complete for every $k \geq 2$.

In this work, we characterize 4-transitive digraphs having a 3-kernel and also 4-transitive digraphs having a 2-kernel. Using the latter result, a proof of the Laborde-Payan-Xuong conjecture for 4-transitive digraphs is given. This conjecture establishes that for every digraph D , an independent set can be found such that it intersects every longest path in D . Also, Seymour's Second Neighborhood Conjecture is verified for 4-transitive digraphs and further problems are proposed.

Keywords: 4-transitive digraph, k -transitive digraph, 3-kernel, k -kernel, Laborde-Payan-Xuong Conjecture.

2010 Mathematics Subject Classification: 05C20.

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Received 14 February 2015
Revised 25 February 2016
Accepted 25 February 2016