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WORM COLORINGS OF PLANAR GRAPHS

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Abstract

Given three planar graphs F, H, and G, an (F, H)-WORM coloring of G is a vertex coloring such that no subgraph isomorphic to F is rainbow and no subgraph isomorphic to H is monochromatic. If G has at least one (F, H)-WORM coloring, then $W^-_{F,H}(G)$ denotes the minimum number of colors in an (F, H)-WORM coloring of G. We show that

(a) $W^-_{F,H}(G) \leq 2$ if $|V(F)| \geq 3$ and H contains a cycle,

(b) $W^-_{F,H}(G) \leq 3$ if $|V(F)| \geq 4$ and H is a forest with $\Delta(H) \geq 3$,

(c) $W^-_{F,H}(G) \leq 4$ if $|V(F)| \geq 5$ and H is a forest with $1 \leq \Delta(H) \leq 2$.

The cases when both F and H are nontrivial paths are more complicated; therefore we consider a relaxation of the original problem. Among others, we prove that any 3-connected plane graph (respectively outerplane graph) admits a 2-coloring such that no facial path on five (respectively four) vertices is monochromatic.

Keywords: plane graph, monochromatic path, rainbow path, WORM coloring, facial coloring.

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