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CLOSED FORMULAE FOR THE STRONG METRIC DIMENSION OF LEXICOGRAPHIC PRODUCT GRAPHS

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Abstract

Given a connected graph G, a vertex $w \in V(G)$ strongly resolves two vertices $u, v \in V(G)$ if there exists some shortest $u - w$ path containing v or some shortest $v - w$ path containing u. A set S of vertices is a strong metric generator for G if every pair of vertices of G is strongly resolved by some vertex of S. The smallest cardinality of a strong metric generator for G is called the strong metric dimension of G. In this paper we obtain several relationships between the strong metric dimension of the lexicographic product of graphs and the strong metric dimension of its factor graphs.

Keywords: strong metric dimension, strong metric basis, strong metric generator, lexicographic product graphs.

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