

RADIO GRACEFUL HAMMING GRAPHS

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Abstract

For $k \in \mathbb{Z}_+$ and G a simple, connected graph, a k -radio labeling $f : V(G) \rightarrow \mathbb{Z}_+$ of G requires all pairs of distinct vertices u and v to satisfy $|f(u) - f(v)| \geq k + 1 - d(u, v)$. We consider k -radio labelings of G when $k = \text{diam}(G)$. In this setting, f is injective; if f is also surjective onto $\{1, 2, \dots, |V(G)|\}$, then f is a *consecutive radio labeling*. Graphs that can be labeled with such a labeling are called *radio graceful*. In this paper, we give two results on the existence of radio graceful Hamming graphs. The main result shows that the Cartesian product of t copies of a complete graph is radio graceful for certain t . Graphs of this form provide infinitely many examples of radio graceful graphs of arbitrary diameter. We also show that these graphs are not radio graceful for large t .

Keywords: radio labeling, radio graceful graph, Hamming graph.

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REFERENCES

- [1] K.F. Benson, M. Porter and M. Tomova, *The radio numbers of all graphs of order n and diameter $n - 2$* , Matematiche (Catania) **68** (2013) 167–190.
- [2] G. Chartrand, D. Erwin, P. Zhang and F. Harary, *Radio labelings of graphs*, Bull. Inst. Combin. Appl. **33** (2001) 77–85.
- [3] G. Chartrand and P. Zhang, *Radio colorings of graphs—a survey*, Int. J. Comput. Appl. Math. **2** (2007) 237–252.
- [4] C. Fernandez, A. Flores, M. Tomova and C. Wyles, *The radio number of gear graphs*, arXiv:0809.2623.
- [5] J. Fiala, J. Kratochvíl and A. Proskurowski, *Distance constrained labeling of precolored trees*, Theoretical Computer Science, Torino, 2001, Lecture Notes in Comput. Sci. (Springer, Berlin, 2001) 285–292.
doi:10.1007/3-540-45446-2_18

- [6] P.C. Fishburn and F.S. Roberts, *No-hole $L(2, 1)$ -colorings*, Discrete Appl. Math. **130** (2003) 513–519.
doi:10.1016/S0166-218X(03)00329-9
- [7] P.C. Fishburn and F.S. Roberts, *Full color theorems for $L(2, 1)$ -colorings*, SIAM J. Discrete Math. **20** (2006) 428–443.
doi:10.1137/S0895480100378562
- [8] J.R. Griggs and R.K. Yeh, *Labelling graphs with a condition at distance 2*, SIAM J. Discrete Math. **5** (1992) 586–595.
doi:10.1137/0405048
- [9] W.K. Hale, *Frequency assignment: theory and applications*, in: Proceedings of the IEEE **68** (1980) 1497–1514.
doi:10.1109/PROC.1980.11899
- [10] X. Li, V. Mak and S. Zhou, *Optimal radio labellings of complete m -ary trees*, Discrete Appl. Math. **158** (2010) 507–515.
doi:10.1016/j.dam.2009.11.014
- [11] D. D.-F. Liu, *Radio number for trees*, Discrete Math. **308** (2008) 1153–1164.
doi:10.1016/j.disc.2007.03.066
- [12] D.D.-F. Liu and M. Xie, *Radio number for square of cycles*, in: Proceedings of the Thirty-Fifth Southeastern International Conference on Combinatorics, Graph Theory and Computing, Congr. Numer. **169** (2004) 101–125.
- [13] D.D.-F. Liu and X. Zhu, *Multilevel distance labelings for paths and cycles*, SIAM J. Discrete Math. **19** (2005) 610–621.
doi:10.1137/S0895480102417768
- [14] P. Martinez, J. Ortiz, M. Tomova and C. Wyels, *Radio numbers for generalized prism graphs*, Discuss. Math. Graph Theory **31** (2011) 45–62.
doi:10.7151/dmgt.1529
- [15] M. Morris-Rivera, M. Tomova, C. Wyels and A. Yeager, *The radio number of $C_n \square C_n$* , Ars Combin. **120** (2015) 7–21.
- [16] L. Saha and P. Panigrahi, *Antipodal number of some powers of cycles*, Discrete Math. **312** (2012) 1550–1557.
doi:10.1016/j.disc.2011.10.032
- [17] L. Saha and P. Panigrahi, *On the radio number of toroidal grids*, Australas. J. Combin. **55** (2013) 273–288.
- [18] L. Saha and P. Panigrahi, *A lower bound for radio k -chromatic number*, Discrete Appl. Math. **192** (2015) 87–100.
doi:10.1016/j.dam.2014.05.004
- [19] B. Sooryanarayana and P. Raghunath, *Radio labeling of cube of a cycle*, Far East J. Appl. Math. **29** (2007) 113–147.
- [20] R. Sweetly and J.P. Joseph, *The radio number of $(W_n : 2)$ graphs*, J. Discrete Math. Sci. Cryptogr. **12** (2009) 729–736.
doi:10.1080/09720529.2009.10698268

- [21] R.K. Yeh, *A survey on labeling graphs with a condition at distance two*, Discrete Math. **306** (2006) 1217–1231.
doi:10.1016/j.disc.2005.11.029

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