

ALL TIGHT DESCRIPTIONS OF 3-STARs IN 3-POLYTOPES WITH GIRTH 5

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Abstract

Lebesgue (1940) proved that every 3-polytope P_5 of girth 5 has a path of three vertices of degree 3. Madaras (2004) refined this by showing that every P_5 has a 3-vertex with two 3-neighbors and the third neighbor of degree at most 4. This description of 3-stars in P_5 s is tight in the sense that no its parameter can be strengthened due to the dodecahedron combined with the existence of a P_5 in which every 3-vertex has a 4-neighbor.

We give another tight description of 3-stars in P_5 s: there is a vertex of degree at most 4 having three 3-neighbors. Furthermore, we show that there are only these two tight descriptions of 3-stars in P_5 s.

Also, we give a tight description of stars with at least three rays in P_5 s and pose a problem of describing all such descriptions. Finally, we prove a structural theorem about P_5 s that might be useful in further research.

Keywords: 3-polytope, planar graph, structure properties, k -star.

2010 Mathematics Subject Classification: 05C15.

¹This work was supported by grants 16-01-00499 and 15-01-05867 of the Russian Foundation for Basic Research and President Grant for Government Support of the Leading Scientific Schools of the Russian Federation NSH-1939.2014.1.

²The author's work was performed as a part of government work "Organizing research" and supported by grant 15-01-05867 of the Russian Foundation for Basic Research.

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Received 22 June 2015

Revised 16 January 2016

Accepted 16 January 2016