

ON THE COMPLEXITY OF REINFORCEMENT IN GRAPHS

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Abstract

We show that the decision problem for p -reinforcement, p -total reinforcement, total restrained reinforcement, and k -rainbow reinforcement are NP-hard for bipartite graphs.

Keywords: domination, total domination, total restrained domination, p -domination, k -rainbow domination, reinforcement, NP-hard.

2010 Mathematics Subject Classification: 05C69.

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Received 14 April 2015

Revised 8 July 2015

Accepted 2 January 2016