

FORBIDDEN SUBGRAPHS FOR HAMILTONICITY OF 1-TOUGH GRAPHS

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Abstract

A graph G is said to be 1-tough if for every vertex cut S of G , the number of components of $G - S$ does not exceed $|S|$. Being 1-tough is an obvious necessary condition for a graph to be hamiltonian, but it is not sufficient in general. We study the problem of characterizing all graphs H such that every 1-tough H -free graph is hamiltonian. We almost obtain a complete solution to this problem, leaving $H = K_1 \cup P_4$ as the only open case.

Keywords: forbidden subgraph, 1-tough graph, H -free graph, hamiltonian graph.

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