Discussiones Mathematicae
Graph Theory 36 (2016) 759–772
doi:10.7151/dmgt.1891

K_3-WORM COLORINGS OF GRAPHS: LOWER CHROMATIC NUMBER AND GAPS IN THE CHROMATIC SPECTRUM

Csilla Bujtás and Zsolt Tuza

Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences, Budapest, Hungary
Department of Computer Science and Systems Technology
University of Pannonia, Veszprém, Hungary

e-mail: bujtas@dcs.uni-pannon.hu
tuza@dcs.uni-pannon.hu

Abstract

A K_3-WORM coloring of a graph G is an assignment of colors to the vertices in such a way that the vertices of each K_3-subgraph of G get precisely two colors. We study graphs G which admit at least one such coloring. We disprove a conjecture of Goddard *et al.* [Congr. Numer. 219 (2014) 161–173] by proving that for every integer $k \geq 3$ there exists a K_3-WORM-colorable graph in which the minimum number of colors is exactly k. There also exist K_3-WORM colorable graphs which have a K_3-WORM coloring with two colors and also with k colors but no coloring with any of $3, \ldots, k-1$ colors. We also prove that it is NP-hard to determine the minimum number of colors, and NP-complete to decide k-colorability for every $k \geq 2$ (and remains intractable even for graphs of maximum degree 9 if $k = 3$). On the other hand, we prove positive results for d-degenerate graphs with small d, also including planar graphs.

Keywords: WORM coloring, lower chromatic number, feasible set, gap in the chromatic spectrum.

2010 Mathematics Subject Classification: 05C15.

References

[1] S. Arumugam, B.K. Jose, Cs. Bujtás and Zs. Tuza, *Equality of domination and transversal numbers in hypergraphs*, Discrete Appl. Math. **161** (2013) 1859–1867. doi:10.1016/j.dam.2013.02.009

[2] G. Bacsó, Cs. Bujtás, Zs. Tuza and V. Voloshin, *New challenges in the theory of hypergraph coloring*, in: Advances in Discrete Mathematics and Applications, Ramanujan Mathematical Society Lecture Notes Series **13** (2010) 45–57.

[3] Cs. Bujtás, E. Sampathkumar, Zs. Tuza, Ch. Dominic and L. Pushpalatha, *Vertex coloring without large polychromatic stars*, Discrete Math. **312** (2012) 2102–2108. doi:10.1016/j.disc.2011.04.013

[4] Cs. Bujtás, E. Sampathkumar, Zs. Tuza, M.S. Subramanya and Ch. Dominic, 3-*consecutive C-colorings of graphs*, Discuss. Math. Graph Theory **30** (2010) 393–405. doi:10.7151/dmgt.1502

[5] Cs. Bujtás and Zs. Tuza, *Maximum number of colors: C-coloring and related problems*, J. Geom. **101** (2011) 83–97. doi:10.1007/s00022-011-0082-2

[6] Cs. Bujtás and Zs. Tuza, K_n-*WORM colorings of graphs: Feasible sets and upper chromatic number*, manuscript in preparation (2015).

[7] Cs. Bujtás, Zs. Tuza and V.I. Voloshin, *Hypergraph colouring*, Chapter 11 in: L.W. Beineke and R.J. Wilson, (Eds.), Topics in Chromatic Graph Theory, Encyclopedia of Mathematics and Its Applications **156** (Cambridge University Press, 2014), 230–254.

[8] W. Goddard, K. Wash and H. Xu, *WORM colorings forbidding cycles or cliques*, Congr. Numer. **219** (2014) 161–173.

[9] W. Goddard, K. Wash and H. Xu, *WORM colorings*, Discuss. Math. Graph Theory **35** (2015) 571–584. doi:10.7151/dmgt.1814

[10] T.R. Jensen and B. Toft, Graph Coloring Problems (Wiley-Interscience, 1995).

[11] A. Kündgen and R. Ramamurthi, *Coloring face-hypergraphs of graphs on surfaces*, J. Combin. Theory Ser. B **85** (2002) 307–337. doi:10.1006/jctb.2001.2107

[12] L. Lovász, *Coverings and colorings of hypergraphs*, Congr. Numer. **8** (1973) 3–12.

[13] D. Marx, *The complexity of chromatic strength and chromatic edge strength*, Comput. Complexity **14** (2006) 308–340. doi:10.1007/s00037-005-0201-2

[14] F. Maffray and M. Preissmann, *On the NP-completeness of the k-colorability problem for triangle-free graphs*, Discrete Math. **162** (1996) 313–317. doi:10.1016/S0012-365X(97)89267-9

[15] K. Ozeki, private communication, June 2015.

[16] V.I. Voloshin, *The mixed hypergraphs*, Comput. Sci. J. Moldova **1** (1993) 45–52.

[17] V.I. Voloshin, *On the upper chromatic number of a hypergraph*, Australas. J. Combin. **11** (1995) 25–45.

[18] V.I. Voloshin, Coloring Mixed Hypergraphs: Theory, Algorithms and Applications (Fields Institute Monographs 17, Amer. Math. Soc., 2002).

[19] V.I. Voloshin, *Mixed hypergraph coloring web site.* http://spectrum.troy.edu/voloshin/mh.html

[20] V.I. Voloshin, private communication, November 2013.

Received 8 September 2015
Revised 27 November 2015
Accepted 27 November 2015