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K_3-WORM COLORINGS OF GRAPHS: LOWER CHROMATIC NUMBER AND GAPS IN THE CHROMATIC SPECTRUM

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Abstract

A K_3-WORM coloring of a graph G is an assignment of colors to the vertices in such a way that the vertices of each K_3-subgraph of G get precisely two colors. We study graphs G which admit at least one such coloring. We disprove a conjecture of Goddard *et al.* [Congr. Numer. 219 (2014) 161–173] by proving that for every integer $k \geq 3$ there exists a K_3-WORM-colorable graph in which the minimum number of colors is exactly k. There also exist K_3-WORM colorable graphs which have a K_3-WORM coloring with two colors and also with k colors but no coloring with any of $3, \ldots, k-1$ colors. We also prove that it is NP-hard to determine the minimum number of colors, and NP-complete to decide k-colorability for every $k \geq 2$ (and remains intractable even for graphs of maximum degree 9 if $k = 3$). On the other hand, we prove positive results for d-degenerate graphs with small d, also including planar graphs.

Keywords: WORM coloring, lower chromatic number, feasible set, gap in the chromatic spectrum.

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