Discussiones Mathematicae
Graph Theory 36 (2016) 695–707
doi:10.7151/dmgt.1890

NORDHAUS-GADDUM-TYPE RESULTS FOR RESISTANCE DISTANCE-BASED GRAPH INVARIANTS

Kinkar Ch. Das

Department of Mathematics
Sungkyunkwan University
Suwon 440–746, Republic of Korea

e-mail: kinkardas2003@googlemail.com

Yujun Yang[1]

School of Mathematics and Information Science
Yantai University
Yantai, Shandong, 264005, PR China

e-mail: yangyj@yahoo.com

AND

Kexiang Xu

College of Science
Nanjing University of Aeronautics & Astronautics
Nanjing, Jiangsu, 210016, PR China

e-mail: kexxu1221@126.com

Abstract

Two decades ago, resistance distance was introduced to characterize "chemical distance" in (molecular) graphs. In this paper, we consider three resistance distance-based graph invariants, namely, the Kirchhoff index, the additive degree-Kirchhoff index, and the multiplicative degree-Kirchhoff index. Some Nordhaus–Gaddum-type results for these three molecular structure descriptors are obtained. In addition, a relation between these Kirchhoffian indices is established.

Keywords: resistance distance, Kirchhoff index, additive degree-Kirchhoff index, multiplicative degree-Kirchhoff index, Nordhaus-Gaddum-type result.

2010 Mathematics Subject Classification: 05C12, 05C90.

[1]Corresponding author.

References

[1] D. Bonchev, A.T. Balaban, X. Liu and D.J. Klein, *Molecular cyclicity and centricity of polycyclic graphs. I. Cyclicity based on resistance distances or reciprocal distances*, Int. J. Quantum Chem. **50** (1994) 1–20.
doi:10.1002/qua.560500102

[2] H. Chen and F. Zhang, *Resistance distance and the normalized Laplacian spectrum*, Discrete Appl. Math. **155** (2007) 654–661.
doi:10.1016/j.dam.2006.09.008

[3] H. Chen and F. Zhang, *Resistance distance local rules*, J. Math. Chem. **44** (2008) 405–417.
doi:10.1007/s10910-007-9317-8

[4] P. Dankelmann, H.C. Swart and P. van den Berg, *Diameter and inverse degree*, Discrete Math. **308** (2008) 670–673.
doi:10.1016/j.disc.2007.07.053

[5] K.Ch. Das, I. Gutman and B. Zhou, *New upper bounds on Zagreb indices*, J. Math. Chem. **46** (2009) 514–521.
doi:10.1007/s10910-008-9475-3

[6] R.M. Foster, *The average impedance of an electrical network*, in: Contributions to Applied Mechanics (Edwards Bros., Michigan, Ann Arbor, 1949) 333–340.

[7] I. Gutman, L. Feng and G. Yu, *Degree resistance distance of unicyclic graphs*, Trans. Comb. **1** (2012) 27–40.

[8] I. Gutman and B. Mohar, *The Quasi-Wiener and the Kirchhoff indices coincide*, J. Chem. Inf. Comput. Sci. **36** (1996) 982–985.
doi:10.1021/ci960007t

[9] D.J. Klein, *Graph geometry, graph metrics, & Wiener*, MATCH Commun. Math. Comput. Chem. **35** (1997) 7–27.

[10] D.J. Klein, *Centrality measure in graphs*, J. Math. Chem. **47** (2010) 1209–1223.
doi:10.1007/s10910-009-9635-0

[11] D.J. Klein and O. Ivanciuc, *Graph cyclicity, excess conductance, and resistance deficit*, J. Math. Chem. **30** (2001) 271–287.
doi:10.1023/A:1015119609980

[12] D.J. Klein and M. Randić, *Resistance distance*, J. Math. Chem. **12** (1993) 81–95.
doi:10.1007/BF01164627

[13] D.J. Klein and H.-Y. Zhu, *Distances and volumina for graphs*, J. Math. Chem. **23** (1998) 179–195.
doi:10.1023/A:1019108905697

[14] E.A. Nordhaus and J.W. Gaddum, *On complementary graphs*, Amer. Math. Monthly **63** (1956) 175–177.
doi:10.2307/2306658

[15] J.L. Palacios and J.M. Renom, *Another look at the degree-Kirchhoff index*, Int. J. Quantum Chem. **111** (2011) 3453–3455.
doi:10.1002/qua.22725

[16] H. Wiener, *Structural determination of paraffin boiling points*, J. Amer. Chem. Soc. **69** (1947) 17–20.
doi:10.1021/ja01193a005

[17] W. Xiao and I. Gutman, *Resistance distance and Laplacian spectrum*, Theor. Chem. Acc. **110** (2003) 284–289.
doi:10.1007/s00214-003-0460-4

[18] Y. Yang, *Relations between resistance distances of a graph and its complement or its contraction*, Croat. Chem. Acta **87** (2014) 61–68.
doi:10.5562/cca2318

[19] Y. Yang, H. Zhang and D.J. Klein, *New Nardhaus-Gaddum-type results for the Kirchhoff index*, J. Math. Chem. **49** (2011) 1587–1598.
doi:10.1007/s10910-011-9845-0

[20] B. Zhou and N. Trinajstić, *A note on Kirchhoff index*, Chem. Phys. Lett. **455** (2008) 120–123.
doi:10.1016/j.cplett.2008.02.060

[21] B. Zhou and N. Trinajstić, *On resistance-distance and Kirchhoff index*, J. Math. Chem. **46** (2009) 283–289.
doi:10.1007/s10910-008-9459-3

[22] H.-Y. Zhu, D.J. Klein and I. Lukovits, *Extensions of the Wiener number*, J. Chem. Inf. Comput. Sci. **36** (1996) 420–428.
doi:10.1021/ci950116s

Received 11 March 2015
Revised 31 October 2015
Accepted 31 October 2015