

## NORDHAUS-GADDUM-TYPE RESULTS FOR RESISTANCE DISTANCE-BASED GRAPH INVARIANTS

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### Abstract

Two decades ago, resistance distance was introduced to characterize “chemical distance” in (molecular) graphs. In this paper, we consider three resistance distance-based graph invariants, namely, the Kirchhoff index, the additive degree-Kirchhoff index, and the multiplicative degree-Kirchhoff index. Some Nordhaus–Gaddum-type results for these three molecular structure descriptors are obtained. In addition, a relation between these Kirchhoffian indices is established.

**Keywords:** resistance distance, Kirchhoff index, additive degree-Kirchhoff index, multiplicative degree-Kirchhoff index, Nordhaus-Gaddum-type result.

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