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THE VERTEX-RAINBOW INDEX OF A GRAPH

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Abstract

The k-rainbow index $\mathrm{rx}_k(G)$ of a connected graph G was introduced by Chartrand, Okamoto and Zhang in 2010. As a natural counterpart of the k-rainbow index, we introduce the concept of k-vertex-rainbow index $\mathrm{rvx}_k(G)$ in this paper. In this paper, sharp upper and lower bounds of $\mathrm{rvx}_k(G)$ are given for a connected graph G of order n, that is, $0 \leq \mathrm{rvx}_k(G) \leq n-2$. We obtain Nordhaus-Gaddum results for 3-vertex-rainbow index of a graph G of order n, and show that $\mathrm{rvx}_3(G) + \mathrm{rvx}_3(\overline{G}) = 4$ for $n = 4$ and $2 \leq \mathrm{rvx}_3(G) + \mathrm{rvx}_3(\overline{G}) \leq n-1$ for $n \geq 5$. Let $t(n,k,\ell)$ denote the minimal size of a connected graph G of order n with $\mathrm{rvx}_k(G) \leq \ell$, where $2 \leq \ell \leq n-2$ and $2 \leq k \leq n$. Upper and lower bounds on $t(n,k,\ell)$ are also obtained.

Keywords: vertex-coloring, connectivity, vertex-rainbow S-tree, vertex-rainbow index, Nordhaus-Gaddum type.

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