

INTEGRAL CAYLEY SUM GRAPHS AND GROUPS

XUANLONG MA^{1,2} AND KAISHUN WANG²

¹*College of Mathematics and Information Science
Guangxi University, Nanning 530004, China*

²*Sch. Math. Sci. & Lab. Math. Com. Sys.
Beijing Normal University
Beijing 100875, China*

e-mail: xuanlma@mail.bnu.edu.cn
wangks@bnu.edu.cn

Abstract

For any positive integer k , let $\mathcal{A}_k$ denote the set of finite abelian groups G such that for any subgroup H of G all Cayley sum graphs $\text{CayS}(H, S)$ are integral if $|S| = k$. A finite abelian group G is called Cayley sum integral if for any subgroup H of G all Cayley sum graphs on H are integral. In this paper, the classes $\mathcal{A}_2$ and $\mathcal{A}_3$ are classified. As an application, we determine all finite Cayley sum integral groups.

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