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INTEGRAL CAYLEY SUM GRAPHS AND GROUPS

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Abstract

For any positive integer k, let $\mathcal{A}_k$ denote the set of finite abelian groups G such that for any subgroup H of G all Cayley sum graphs $\mathrm{CayS}(H, S)$ are integral if $|S| = k$. A finite abelian group G is called Cayley sum integral if for any subgroup H of G all Cayley sum graphs on H are integral. In this paper, the classes $\mathcal{A}_2$ and $\mathcal{A}_3$ are classified. As an application, we determine all finite Cayley sum integral groups.

Keywords: Cayley sum graph, integral graph, Cayley sum integral group.

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References

[1] A. Abdollahi and M. Jazaeri, *Groups all of whose undirected Cayley graphs are integral*, European J. Combin. **38** (2014) 102–109.
doi:10.1016/j.ejc.2013.11.007

[2] A. Abdollahi and E. Vatandoost, *Which Cayley graphs are integral?*, Electron. J. Combin. **16** (2009) #R122.

[3] A. Ahmady, J.P. Bell and B. Mohar, *Integral Cayley graphs and groups*, SIAM J. Discrete Math. **28** (2014) 685–701.
doi:10.1137/130925487

[4] N. Alon, *Large sets in finite fields are sumsets*, J. Number Theory **126** (2007) 110–118.
doi:10.1016/j.jnt.2006.11.007

[5] M. Amooshahi and B. Taeri, *On Cayley sum graphs of non-Abelian groups*, Graphs Combin. **32** (2016) 17–29.
doi:10.1007/s00373-015-1535-4

[6] A.E. Brouwer and W.H. Haemers, Spectra of Graphs (Springer-Verlag, New York, 2012).
doi:10.1007/978-1-4614-1939-6

[7] F.C. Bussemaker and D.M. Cvetković, *There are exactly* 13 *connected, cubic, integral graphs*, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **544–576** (1976) 43–48.

[8] B. Cheyne, V. Gupta and C. Wheeler, *Hamilton cycles in addition graphs*, Rose-Hulman Undergrad. Math J. **4** (2003) 1–17.

[9] F.R.K. Chung, *Diameters and eigenvalues*, J. Amer. Math. Soc. **2** (1989) 187–196.
doi:10.1090/S0894-0347-1989-0965008-X

[10] M. DeVos, L. Goddyn, B. Mohar and R. Šámal, *Cayley sum graphs and eigenvalues of* $(3,6)$*-fullerenes*, J. Combin. Theory Ser. B **99** (2009) 358–369.
doi:10.1016/j.jctb.2008.08.005

[11] I. Estélyi and I. Kovács, *On groups all of whose undirected Cayley graphs of bounded valency are integral*, Electron. J. Combin. **21** (2014) #P4.45.

[12] B.J. Green, *Counting sets with small sumset, and the clique number of random Cayley graphs*, Combinatorica **25** (2005) 307–326.
doi:s00493-005-0018-2

[13] F. Harary and A.J. Schwenk, *Which graphs have integral spectra*? in: R.A. Bari and F. Harary (Eds.), Graphs and Combinatorics, Lecture Notes in Math. **406** (Springer, 1974) 45–51.
doi:10.1007/bfb0066434

[14] W. Klotz and T. Sander, *Integral Cayley graphs over abelian groups*, Electron. J. Combin. **17** (2010) #R81.

[15] X. Ma and K. Wang, *On finite groups all of whose cubic Cayley graphs are integral*, J. Algebra Appl. **15** (2016) 1650105, 10 pp.
doi:10.1142/S021949881650105X

[16] A.J. Schwenk, *Exactly thirteen connected cubic graphs have integral spectra*, in: Y. Alavi and D.R. Lick (Eds.), Theory and Applications of Graphs, Lecture Notes in Math. **642** (Springer, 1978) 516–533.
doi:10.1007/BFb0070407

[17] W. So, *Integral circulant graphs*, Discrete Math. **306** (2006) 153–158.
doi:10.1016/j.disc.2005.11.006

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