Discussiones Mathematicae
Graph Theory 36 (2016) 709–722
doi:10.7151/dmgt.1884

SUM LIST EDGE COLORINGS OF GRAPHS

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Abstract

Let $G = (V, E)$ be a simple graph and for every edge $e \in E$ let $L(e)$ be a set (list) of available colors. The graph G is called *L-edge colorable* if there is a proper edge coloring c of G with $c(e) \in L(e)$ for all $e \in E$. A function $f : E \to \mathbb{N}$ is called an *edge choice function* of G and G is said to be *f-edge choosable* if G is L-edge colorable for every list assignment L with $|L(e)| = f(e)$ for all $e \in E$. Set $\text{size}(f) = \sum_{e \in E} f(e)$ and define the *sum choice index* $\chi'_{sc}(G)$ as the minimum of $\text{size}(f)$ over all edge choice functions f of G.

There exists a greedy coloring of the edges of G which leads to the upper bound $\chi'_{sc}(G) \le \frac{1}{2} \sum_{v \in V} d(v)^2$. A graph is called *sec-greedy* if its sum choice index equals this upper bound.

We present some general results on the sum choice index of graphs including a lower bound and we determine this index for several classes of graphs. Moreover, we present classes of *sec*-greedy graphs as well as all such graphs of order at most 5.

Keywords: sum list edge coloring, sum choice index, sum list coloring, sum choice number, choice function, line graph.

2010 Mathematics Subject Classification: 05C15.

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Received 18 December 2014
Revised 2 November 2015
Accepted 2 November 2015