

## TRIANGLE DECOMPOSITIONS OF PLANAR GRAPHS

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### Abstract

A multigraph  $G$  is triangle decomposable if its edge set can be partitioned into subsets, each of which induces a triangle of  $G$ , and rationally triangle decomposable if its triangles can be assigned rational weights such that for each edge  $e$  of  $G$ , the sum of the weights of the triangles that contain  $e$  equals 1.

We present a necessary and sufficient condition for a planar multigraph to be triangle decomposable. We also show that if a simple planar graph is rationally triangle decomposable, then it has such a decomposition using only weights 0, 1 and  $\frac{1}{2}$ . This result provides a characterization of rationally triangle decomposable simple planar graphs. Finally, if  $G$  is a multigraph with  $K_4$  as underlying graph, we give necessary and sufficient conditions on the multiplicities of its edges for  $G$  to be triangle and rationally triangle decomposable.

**Keywords:** planar graphs, triangle decompositions, rational triangle decompositions.

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