# TRIANGLE DECOMPOSITIONS OF PLANAR GRAPHS 

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#### Abstract

A multigraph $G$ is triangle decomposable if its edge set can be partitioned into subsets, each of which induces a triangle of $G$, and rationally triangle decomposable if its triangles can be assigned rational weights such that for each edge $e$ of $G$, the sum of the weights of the triangles that contain $e$ equals 1 .

We present a necessary and sufficient condition for a planar multigraph to be triangle decomposable. We also show that if a simple planar graph is rationally triangle decomposable, then it has such a decomposition using only weights 0,1 and $\frac{1}{2}$. This result provides a characterization of rationally triangle decomposable simple planar graphs. Finally, if $G$ is a multigraph with $K_{4}$ as underlying graph, we give necessary and sufficient conditions on the multiplicities of its edges for $G$ to be triangle and rationally triangle decomposable.


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