THE SMALLEST NON-AUTOGRAPH

Benjamin S. Baumer
Program in Statistical & Data Sciences
Smith College
e-mail: bbaumer@smith.edu

Yijin Wei
Department of Mathematics and Statistics
Smith College
e-mail: ywei@smith.edu

AND

Gary S. Bloom
Department of Computer Science
City College

Abstract

Suppose that $G$ is a simple, vertex-labeled graph and that $S$ is a multiset. Then if there exists a one-to-one mapping between the elements of $S$ and the vertices of $G$, such that edges in $G$ exist if and only if the absolute difference of the corresponding vertex labels exist in $S$, then $G$ is an autograph, and $S$ is a signature for $G$. While it is known that many common families of graphs are autographs, and that infinitely many graphs are not autographs, a non-autograph has never been exhibited. In this paper, we identify the smallest non-autograph: a graph with 6 vertices and 11 edges. Furthermore, we demonstrate that the infinite family of graphs on $n$ vertices consisting of the complement of two non-intersecting cycles contains only non-autographs for $n \geq 8$.

Keywords: graph labeling, difference graphs, autographs, monographs.

2010 Mathematics Subject Classification: Primary 05C78; Secondary 05C60.
References


Received 29 May 2015
Accepted 25 September 2015