# THE SMALLEST NON-AUTOGRAPH 

Benjamin S. Baumer<br>Program in Statistical \& Data Sciences<br>Smith College<br>e-mail: bbaumer@smith.edu<br>Yijin Wei<br>Department of Mathematics and Statistics<br>Smith College<br>e-mail: ywei@smith.edu

AND

Gary S. Bloom<br>Department of Computer Science<br>City College


#### Abstract

Suppose that $G$ is a simple, vertex-labeled graph and that $S$ is a multiset. Then if there exists a one-to-one mapping between the elements of $S$ and the vertices of $G$, such that edges in $G$ exist if and only if the absolute difference of the corresponding vertex labels exist in $S$, then $G$ is an autograph, and $S$ is a signature for $G$. While it is known that many common families of graphs are autographs, and that infinitely many graphs are not autographs, a non-autograph has never been exhibited. In this paper, we identify the smallest non-autograph: a graph with 6 vertices and 11 edges. Furthermore, we demonstrate that the infinite family of graphs on $n$ vertices consisting of the complement of two non-intersecting cycles contains only non-autographs for $n \geq 8$.


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