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THE SMALLEST NON-AUTOGRAPH

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Abstract

Suppose that G is a simple, vertex-labeled graph and that S is a multiset. Then if there exists a one-to-one mapping between the elements of S and the vertices of G, such that edges in G exist if and only if the absolute difference of the corresponding vertex labels exist in S, then G is an *autograph*, and S is a *signature* for G. While it is known that many common families of graphs are autographs, and that infinitely many graphs are not autographs, a non-autograph has never been exhibited. In this paper, we identify the smallest non-autograph: a graph with 6 vertices and 11 edges. Furthermore, we demonstrate that the infinite family of graphs on n vertices consisting of the complement of two non-intersecting cycles contains only non-autographs for $n \geq 8$.

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