

## VARIOUS BOUNDS FOR LIAR'S DOMINATION NUMBER

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### Abstract

Let  $G = (V, E)$  be a graph. A set  $S \subseteq V$  is a dominating set if  $\bigcup_{v \in S} N[v] = V$ , where  $N[v]$  is the closed neighborhood of  $v$ . Let  $L \subseteq V$  be a dominating set, and let  $v$  be a designated vertex in  $V$  (an intruder vertex). Each vertex in  $L \cap N[v]$  can report that  $v$  is the location of the intruder, but (at most) one  $x \in L \cap N[v]$  can report any  $w \in N[x]$  as the intruder location or  $x$  can indicate that there is no intruder in  $N[x]$ . A dominating set  $L$  is called a liar's dominating set if every  $v \in V(G)$  can be correctly identified as an intruder location under these restrictions. The minimum cardinality of a liar's dominating set is called the liar's domination number, and is denoted by  $\gamma_{LR}(G)$ . In this paper, we present sharp bounds for the liar's domination number in terms of the diameter, the girth and clique covering number of a graph. We present two Nordhaus-Gaddum type relations for  $\gamma_{LR}(G)$ , and study liar's dominating set sensitivity versus edge-connectivity. We also present various bounds for the liar's domination component number, that is, the maximum number of components over all minimum liar's dominating sets.

**Keywords:** liar's domination, diameter, regular graph, Nordhaus-Gaddum.

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