

THE RYJÁČEK CLOSURE AND A FORBIDDEN SUBGRAPH

AKIRA SAITO

Department of Information Science
Nihon University
Sakurajosui 3-25-40
Setagaya-Ku, Tokyo 156-8550, Japan
e-mail: asaito@chs.nihon-u.ac.jp

AND

LIMING XIONG

School of Mathematics and Statistics
Beijing Institute of Technology
Beijing 100081, China
and
Beijing Key Laboratory on MCAACI
Beijing Institute of Technology
Beijing 100081, China
e-mail: lmxiong@bit.edu.cn

Abstract

The Ryjáček closure is a powerful tool in the study of Hamiltonian properties of claw-free graphs. Because of its usefulness, we may hope to use it in the classes of graphs defined by another forbidden subgraph. In this note, we give a negative answer to this hope, and show that the claw is the only forbidden subgraph that produces non-trivial results on Hamiltonicity by the use of the Ryjáček closure.

Keywords: closure, claw-free graph, Hamiltonian graph, perfect matching, traceable graph.

2010 Mathematics Subject Classification: 05C45, 05C70.

REFERENCES

- [1] G. Chartrand, L. Lesniak and P. Zhang, *Graphs & Digraphs* (5th Ed.) (Chapman and Hall/CRC, Boca Raton, Florida, USA, 2010).

- [2] M. Jünger, W.R. Pulleyblank and G. Reinelt, *On partitioning the edges of graphs into connected subgraphs*, J. Graph Theory **9** (1985) 539–549.
doi:10.1002/jgt.3190090416
- [3] M. Las Vergnas, *A note on matchings in graphs*, Colloque sur la Théorie des Graphes, Cahiers Centre Études Rech. Opér. **17** (1975) 257–260.
- [4] M.M. Matthews and D.P. Sumner, *Hamiltonian results in $K_{1,3}$ -free graphs*, J. Graph Theory **8** (1984) 139–146.
doi:10.1002/jgt.3190080116
- [5] D.J. Oberly and D.P. Sumner, *Every connected, locally connected nontrivial graph with no induced claw is Hamiltonian*, J. Graph Theory **3** (1979) 351–356.
doi:10.1002/jgt.3190030405
- [6] M.D. Plummer and A. Saito, *Forbidden subgraphs and bounds on the size of a maximum matching*, J. Graph Theory **50** (2005) 1–12.
doi:10.1002/jgt.20087
- [7] Z. Ryjáček, *On a closure concept in claw-free graphs*, J. Combin. Theory Ser. B **70** (1997) 217–224.
doi:10.1006/jctb.1996.1732
- [8] D.P. Sumner, *1-factors and antifactor sets*, J. Lond. Math. Soc. (2) **13** (1976) 351–359.
doi:10.1112/jlms/s2-13.2.351
- [9] C. Thomassen, *Reflections on graph theory*, J. Graph Theory **10** (1986) 309–324.
doi:10.1002/jgt.3190100308

Received 29 May 2015
 Revised 30 September 2015
 Accepted 30 September 2015