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THE RYJÁČEK CLOSURE AND A FORBIDDEN SUBGRAPH

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Abstract

The Ryjáček closure is a powerful tool in the study of Hamiltonian properties of claw-free graphs. Because of its usefulness, we may hope to use it in the classes of graphs defined by another forbidden subgraph. In this note, we give a negative answer to this hope, and show that the claw is the only forbidden subgraph that produces non-trivial results on Hamiltonicity by the use of the Ryjáček closure.

Keywords: closure, claw-free graph, Hamiltonian graph, perfect matching, traceable graph.

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