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ON LONGEST CYCLES IN ESSENTIALLY 4-CONNECTED PLANAR GRAPHS

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Abstract

A planar 3-connected graph G is essentially 4-connected if, for any 3-separator S of G, one component of the graph obtained from G by removing S is a single vertex. Jackson and Wormald proved that an essentially 4-connected planar graph on n vertices contains a cycle C such that $|V(C)| \geq \frac{2n+4}{5}$. For a cubic essentially 4-connected planar graph G, Grünbaum with Malkevitch, and Zhang showed that G has a cycle on at least $\frac{3}{4}n$ vertices. In the present paper the result of Jackson and Wormald is improved. Moreover, new lower bounds on the length of a longest cycle of G are presented if G is an essentially 4-connected planar graph of maximum degree 4 or G is an essentially 4-connected maximal planar graph.

Keywords: planar graph, longest cycle.

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