

ON LONGEST CYCLES IN ESSENTIALLY 4-CONNECTED PLANAR GRAPHS

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Abstract

A planar 3-connected graph G is essentially 4-connected if, for any 3-separator S of G , one component of the graph obtained from G by removing S is a single vertex. Jackson and Wormald proved that an essentially 4-connected planar graph on n vertices contains a cycle C such that $|V(C)| \geq \frac{2n+4}{5}$. For a cubic essentially 4-connected planar graph G , Grünbaum with Malkevitch, and Zhang showed that G has a cycle on at least $\frac{3}{4}n$ vertices. In the present paper the result of Jackson and Wormald is improved. Moreover, new lower bounds on the length of a longest cycle of G are presented if G is an essentially 4-connected planar graph of maximum degree 4 or G is an essentially 4-connected maximal planar graph.

Keywords: planar graph, longest cycle.

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REFERENCES

- [1] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Springer, 2008).
- [2] I. Fabrici, J. Harant and S. Jendroľ, *Paths of low weight in planar graphs*, *Discuss. Math. Graph Theory* **28** (2008) 121–135.
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- [3] B. Grünbaum and J. Malkevitch, *Pairs of edge-disjoint Hamilton circuits*, Aequationes Math. **14** (1976) 191–196.
doi:10.1007/BF01836218
- [4] B. Jackson and N.C. Wormald, *Longest cycles in 3-connected planar graphs*, J. Combin. Theory Ser. B **54** (1992) 291–321.
doi:10.1016/0095-8956(92)90058-6
- [5] D.P. Sanders, *On paths in planar graphs*, J. Graph Theory **24** (1997) 341–345.
doi:10.1002/(SICI)1097-0118(199704)24:4<341::AID-JGT6>3.0.CO;2-O
- [6] C. Thomassen, *A theorem on paths in planar graphs*, J. Graph Theory **7** (1983) 169–176.
doi:10.1002/jgt.3190070205
- [7] W.T. Tutte, *A theorem on planar graphs*, Trans. Amer. Math. Soc. **82** (1956) 99–116.
doi:10.1090/S0002-9947-1956-0081471-8
- [8] C.-Q. Zhang, *Longest cycles and their chords*, J. Graph Theory **11** (1987) 521–529.
doi:10.1002/jgt.3190110409

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