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NEW BOUNDS ON THE SIGNED TOTAL DOMINATION NUMBER OF GRAPHS

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Abstract

In this paper, we study the signed total domination number in graphs and present new sharp lower and upper bounds for this parameter. For example by making use of the classic theorem of Turán [8], we present a sharp lower bound on K_{r+1}-free graphs for $r \geq 2$. Applying the concept of total limited packing we bound the signed total domination number of G with $\delta(G) \geq 3$ from above by $n - 2\left\lfloor \frac{2\rho_o(G)+\delta-3}{2} \right\rfloor$. Also, we prove that $\gamma_{st}(T) \leq n - 2(s - s')$ for any tree T of order n, with s support vertices and s' support vertices of degree two. Moreover, we characterize all trees attaining this bound.

Keywords: open packing, signed total domination number, total limited packing, tuple total domination number.

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