

LOOSENESS AND INDEPENDENCE NUMBER OF TRIANGULATIONS ON CLOSED SURFACES

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Abstract

The *looseness* of a triangulation G on a closed surface F^2 , denoted by $\xi(G)$, is defined as the minimum number k such that for any surjection $c : V(G) \rightarrow \{1, 2, \dots, k+3\}$, there is a face uvw of G with $c(u), c(v)$ and $c(w)$ all distinct. We shall bound $\xi(G)$ for triangulations G on closed surfaces by the independence number of G denoted by $\alpha(G)$. In particular, for a triangulation G on the sphere, we have

$$\xi(G) \leq \frac{11\alpha(G) - 10}{6}$$

and this bound is sharp. For a triangulation G on a non-spherical surface F^2 , we have

$$\xi(G) \leq 2\alpha(G) + l(F^2) - 2,$$

where $l(F^2) = \lfloor (2 - \chi(F^2))/2 \rfloor$ with Euler characteristic $\chi(F^2)$.

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