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LOOSENESS AND INDEPENDENCE NUMBER OF TRIANGULATIONS ON CLOSED SURFACES

Atsuhiro Nakamoto, Seiya Negami

Graduate School of Environment and Information Science
Yokohama National University
79-7 Tokiwadai, Hodogaya-Ku, Yokohama 240-8501, Japan

e-mail: nakamoto@ynu.ac.jp, negami@ynu.ac.jp

Kyoji Ohba

Yonago National College of Technology
Yonago, Tottori 683-8502, Japan

e-mail: ooba@yonago-k.ac.jp

and

Yusuke Suzuki

Department of Mathematics
Niigata University
8050 Ikarashi 2-no-cho, Nishi-ku, Niigata, 950-2181, Japan

e-mail: y-suzuki@math.sc.niigata-u.ac.jp

Abstract

The *looseness* of a triangulation G on a closed surface F^2, denoted by $\xi(G)$, is defined as the minimum number k such that for any surjection $c : V(G) \to \{1, 2, \ldots, k+3\}$, there is a face uvw of G with $c(u), c(v)$ and $c(w)$ all distinct. We shall bound $\xi(G)$ for triangulations G on closed surfaces by the independence number of G denoted by $\alpha(G)$. In particular, for a triangulation G on the sphere, we have

$$\xi(G) \leq \frac{11\alpha(G) - 10}{6}$$

and this bound is sharp. For a triangulation G on a non-spherical surface F^2, we have

$$\xi(G) \leq 2\alpha(G) + l(F^2) - 2,$$

where $l(F^2) = \lfloor (2 - \chi(F^2))/2 \rfloor$ with Euler characteristic $\chi(F^2)$.

Keywords: triangulations, closed surfaces, looseness, k-loosely tight, independence number.

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