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THE STEINER WIENER INDEX OF A GRAPH

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Abstract

The Wiener index $W(G)$ of a connected graph G, introduced by Wiener in 1947, is defined as $W(G) = \sum_{u,v \in V(G)} d(u,v)$ where $d_G(u,v)$ is the distance between vertices u and v of G. The Steiner distance in a graph, introduced by Chartrand *et al.* in 1989, is a natural generalization of the concept of classical graph distance. For a connected graph G of order at least 2 and $S \subseteq V(G)$, the *Steiner distance* $d(S)$ of the vertices of S is the minimum size of a connected subgraph whose vertex set is S. We now introduce the concept of the Steiner Wiener index of a graph. The *Steiner k-Wiener index* $SW_k(G)$ of G is defined by $SW_k(G) = \sum_{\substack{S \subseteq V(G) \\ |S|=k}} d(S)$. Expressions for SW_k for some special graphs are obtained. We also give sharp upper and lower bounds of SW_k of a connected graph, and establish some of its properties in

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the case of trees. An application in chemistry of the Steiner Wiener index is reported in our another paper.

Keywords: distance, Steiner distance, Wiener index, Steiner Wiener k-index.

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