Discussiones Mathematicae
Graph Theory 36 (2016) 439–453
doi:10.7151/dmgt.1867

CHARACTERIZATIONS OF GRAPHS HAVING LARGE PROPER CONNECTION NUMBERS

Chira Lumduanhom[1]

Department of Mathematics
Srinakharinwirot University
Sukhumvit Soi 23, Bangkok, 10110, Thailand

Elliot Laforge and Ping Zhang

Department of Mathematics
Western Michigan University
Kalamazoo, MI 49008, USA

e-mail: ping.zhang@wmich.edu

Abstract

Let G be an edge-colored connected graph. A path P is a proper path in G if no two adjacent edges of P are colored the same. If P is a proper $u-v$ path of length $d(u,v)$, then P is a proper $u-v$ geodesic. An edge coloring c is a proper-path coloring of a connected graph G if every pair u,v of distinct vertices of G are connected by a proper $u-v$ path in G, and c is a strong proper-path coloring if every two vertices u and v are connected by a proper $u-v$ geodesic in G. The minimum number of colors required for a proper-path coloring or strong proper-path coloring of G is called the proper connection number $\mathrm{pc}(G)$ or strong proper connection number $\mathrm{spc}(G)$ of G, respectively. If G is a nontrivial connected graph of size m, then $\mathrm{pc}(G) \leq \mathrm{spc}(G) \leq m$ and $\mathrm{pc}(G) = m$ or $\mathrm{spc}(G) = m$ if and only if G is the star of size m. In this paper, we determine all connected graphs G of size m for which $\mathrm{pc}(G)$ or $\mathrm{spc}(G)$ is $m-1, m-2$ or $m-3$.

Keywords: edge coloring, proper-path coloring, strong proper-path coloring.

2010 Mathematics Subject Classification: 05C15, 05C38, 05C75.

References

[1]Research supported by a New Researcher Grants sponsored by Ministry of Science & Technology, Thailand.

[1] E. Andrews, E. Laforge, C. Lumduanhom and P. Zhang, *On proper-path colorings in graphs*, J. Combin. Math. Combin. Comput., to appear.

[2] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.

[3] G. Chartrand, L. Lesniak and P. Zhang, Graphs & Digraphs, 5th Edition (Chapman & Hall/CRC, Boca Raton, FL, 2010).

[4] G. Chartrand and P. Zhang, Chromatic Graph Theory (Chapman & Hall/CRC, Boca Raton, FL, 2008).
doi:10.1201/9781584888017

[5] A.B. Ericksen, *A matter of security*, Graduating Engineer & Computer Careers (2007) 24–28.

[6] X.L. Li and Y.F. Sun, Rainbow Connections of Graphs (Springer, Boston, MA, 2012).
doi:10.1007/978-1-4614-3119-0

[7] X.L. Li, Y.F. Sun and Y. Zhao, *Characterization of graphs with rainbow connection number* $m-2$ *and* $m-3$, Australas. J. Combin. **60** (2014) 306–313.

Received 15 October 2014
Revised 5 August 2015
Accepted 10 August 2015