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A CHARACTERIZATION OF HYPERGRAPHS WITH LARGE DOMINATION NUMBER

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Abstract

Let $H = (V, E)$ be a hypergraph with vertex set V and edge set E. A dominating set in H is a subset of vertices $D \subseteq V$ such that for every vertex $v \in V \setminus D$ there exists an edge $e \in E$ for which $v \in e$ and $e \cap D \neq \emptyset$. The domination number $\gamma(H)$ is the minimum cardinality of a dominating set in H. It is known [Cs. Bujtás, M.A. Henning and Zs. Tuza, *Transversals and domination in uniform hypergraphs*, European J. Combin. **33** (2012) 62–71] that for $k \geq 5$, if H is a hypergraph of order n and size m with all edges of size at least k and with no isolated vertex, then $\gamma(H) \leq (n + \lfloor (k-3)/2 \rfloor m)/(\lfloor 3(k-1)/2 \rfloor)$. In this paper, we apply a recent result of the authors on hypergraphs with large transversal number [M.A. Henning and C. Löwenstein, *A characterization of hypergraphs that achieve equality in the Chvátal-McDiarmid Theorem*, Discrete Math. **323** (2014) 69–75] to characterize the hypergraphs achieving equality in this bound.

Keywords: domination, transversal, hypergraph.

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