

A NEIGHBORHOOD CONDITION FOR FRACTIONAL ID- $[A, B]$ -FACTOR-CRITICAL GRAPHS

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Abstract

Let G be a graph of order n , and let a and b be two integers with $1 \leq a \leq b$. Let $h : E(G) \rightarrow [0, 1]$ be a function. If $a \leq \sum_{e \ni x} h(e) \leq b$ holds for any $x \in V(G)$, then we call $G[F_h]$ a fractional $[a, b]$ -factor of G with indicator function h , where $F_h = \{e \in E(G) : h(e) > 0\}$. A graph G is fractional independent-set-deletable $[a, b]$ -factor-critical (in short, fractional ID- $[a, b]$ -factor-critical) if $G - I$ has a fractional $[a, b]$ -factor for every independent set I of G . In this paper, it is proved that if $n \geq \frac{(a+2b)(2a+2b-3)+1}{b}$, $\delta(G) \geq \frac{bn}{a+2b} + a$ and $|N_G(x) \cup N_G(y)| \geq \frac{(a+b)n}{a+2b}$ for any two nonadjacent vertices $x, y \in V(G)$, then G is fractional ID- $[a, b]$ -factor-critical. Furthermore, it is shown that this result is best possible in some sense.

Keywords: graph, minimum degree, neighborhood, fractional $[a, b]$ -factor, fractional ID- $[a, b]$ -factor-critical graph.

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