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A NEIGHBORHOOD CONDITION FOR FRACTIONAL ID-[A, B]-FACTOR-CRITICAL GRAPHS

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Abstract

Let G be a graph of order n, and let a and b be two integers with $1 \leq a \leq b$. Let $h : E(G) \to [0, 1]$ be a function. If $a \leq \sum_{e \ni x} h(e) \leq b$ holds for any $x \in V(G)$, then we call $G[F_h]$ a fractional $[a, b]$-factor of G with indicator function h, where $F_h = \{e \in E(G) : h(e) > 0\}$. A graph G is fractional independent-set-deletable $[a, b]$-factor-critical (in short, fractional ID-$[a, b]$-factor-critical) if $G - I$ has a fractional $[a, b]$-factor for every independent set I of G. In this paper, it is proved that if $n \geq \frac{(a+2b)(2a+2b-3)+1}{b}$, $\delta(G) \geq \frac{bn}{a+2b} + a$ and $|N_G(x) \cup N_G(y)| \geq \frac{(a+b)n}{a+2b}$ for any two nonadjacent vertices $x, y \in V(G)$, then G is fractional ID-$[a, b]$-factor-critical. Furthermore, it is shown that this result is best possible in some sense.

Keywords: graph, minimum degree, neighborhood, fractional $[a, b]$-factor, fractional ID-$[a, b]$-factor-critical graph.

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References

[1] S. Zhou, Z. Sun and H. Liu, *A minimum degree condition for fractional ID-$[a,b]$-factor-critical graphs*, Bull. Aust. Math. Soc. **86** (2012) 177–183. doi:10.1017/S0004972711003467

[2] H. Matsuda, *A neighborhood condition for graphs to have $[a,b]$-factors*, Discrete Math. **224** (2000) 289–292. doi:10.1016/S0012-365X(00)00140-0

[3] J. Ekstein, P. Holub, T. Kaiser, L. Xiong and S. Zhang, *Star subdivisions and connected even factors in the square of a graph*, Discrete Math. **312** (2012) 2574–2578. doi:10.1016/j.disc.2011.09.004

[4] H. Lu, *Simplified existence theorems on all fractional $[a,b]$-factors*, Discrete Appl. Math. **161** (2013) 2075–2078. doi:10.1016/j.dam.2013.02.006

[5] S. Zhou, *Independence number, connectivity and (a,b,k)-critical graphs*, Discrete Math. **309** (2009) 4144–4148. doi:10.1016/j.disc.2008.12.013

[6] S. Zhou, *A sufficient condition for graphs to be fractional (k,m)-deleted graphs*, Appl. Math. Lett. **24** (2011) 1533–1538. doi:10.1016/j.aml.2011.03.041

[7] S. Zhou and H. Liu, *Neighborhood conditions and fractional k-factors*, Bull. Malays. Math. Sci. Soc. **32** (2009) 37–45.

[8] K. Kimura, *f-factors, complete-factors, and component-deleted subgraphs*, Discrete Math. **313** (2013) 1452–1463. doi:10.1016/j.disc.2013.03.009

[9] M. Kouider and Z. Lonc, *Stability number and $[a,b]$-factors in graphs*, J. Graph Theory **46** (2004) 254–264. doi:10.1002/jgt.20008

[10] R. Chang, G. Liu and Y. Zhu, *Degree conditions of fractional ID-k-factor-critical graphs*, Bull. Malays. Math. Sci. Soc. **33** (2010) 355–360.

[11] S. Zhou, Q. Bian and J. Wu, *A result on fractional ID-k-factor-critical graphs*, J. Combin. Math. Combin. Comput. **87** (2013) 229–236.

[12] S. Zhou, *Binding numbers for fractional ID-k-factor-critical graphs*, Acta Math. Sin. Engl. Ser. **30** (2014) 181–186. doi:10.1007/s10114-013-1396-9

[13] G. Liu and L. Zhang, *Fractional (g,f)-factors of graphs*, Acta Math. Sci. Ser. B **21** (2001) 541–545.

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