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KERNELS BY MONOCHROMATIC PATHS AND COLOR-PERFECT DIGRAPHS

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Abstract

For a digraph D, $V(D)$ and $A(D)$ will denote the sets of vertices and arcs of D respectively. In an arc-colored digraph, a subset K of $\mathrm{V}(D)$ is said to be *kernel by monochromatic paths* (mp-kernel) if (1) for any two different vertices x, y in N there is no monochromatic directed path between them (N is mp-independent) and (2) for each vertex u in $V(D) \setminus N$ there exists $v \in N$ such that there is a monochromatic directed path from u to v in D (N is mp-absorbent). If every arc in D has a different color, then a kernel by monochromatic paths is said to be a *kernel*. Two associated digraphs to an arc-colored digraph are the closure and the color-class digraph $\mathscr{C}_C(D)$. In this paper we will approach an mp-kernel via the closure of induced subdigraphs of D which have the property of having few colors in their arcs with respect to D. We will introduce the concept of color-perfect digraph and we are going to prove that if D is an arc-colored digraph such that D is a quasi color-perfect digraph and $\mathscr{C}_C(D)$ is not strong, then D has an mp-kernel. Previous interesting results are generalized, as for example Richardson's Theorem.

Keywords: kernel, kernel perfect digraph, kernel by monochromatic paths, color-class digraph, quasi color-perfect digraph, color-perfect digraph.

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