# SOLUTIONS OF SOME $L(2,1)$-COLORING RELATED OPEN PROBLEMS 

Nibedita Mandal and Pratima Panigrahi<br>Department of Mathematics Indian Institute of Technology Kharagpur<br>India<br>e-mail: nibedita.mandal.iitkgp@gmail.com pratima@maths.iitkgp.ernet.in


#### Abstract

An $L(2,1)$-coloring (or labeling) of a graph $G$ is a vertex coloring $f$ : $V(G) \rightarrow Z^{+} \cup\{0\}$ such that $|f(u)-f(v)| \geq 2$ for all edges $u v$ of $G$, and $|f(u)-f(v)| \geq 1$ if $d(u, v)=2$, where $d(u, v)$ is the distance between vertices $u$ and $v$ in $G$. The span of an $L(2,1)$-coloring is the maximum color (or label) assigned by it. The span of a graph $G$ is the smallest integer $\lambda$ such that there exists an $L(2,1)$-coloring of $G$ with span $\lambda$. An $L(2,1)$-coloring of a graph with span equal to the span of the graph is called a span coloring. For an $L(2,1)$-coloring $f$ of a graph $G$ with span $k$, an integer $h$ is a hole in $f$ if $h \in(0, k)$ and there is no vertex $v$ in $G$ such that $f(v)=h$. A no-hole coloring is an $L(2,1)$-coloring with no hole in it. An $L(2,1)$-coloring is irreducible if color of none of the vertices in the graph can be decreased to yield another $L(2,1)$-coloring of the same graph. A graph $G$ is inh-colorable if there exists an irreducible no-hole coloring of $G$. Most of the results obtained in this paper are answers to some problems asked by Laskar et al. [5]. These problems are mainly about relationship between the span and maximum no-hole span of a graph, lower inh-span and upper inh-span of a graph, and the maximum number of holes and minimum number of holes in a span coloring of a graph. We also give some sufficient conditions for a tree and an unicyclic graph to have inh-span $\Delta+1$.


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