

SOLUTIONS OF SOME $L(2, 1)$ -COLORING RELATED OPEN PROBLEMS

NIBEDITA MANDAL AND PRATIMA PANIGRAHI

*Department of Mathematics
Indian Institute of Technology Kharagpur
India*

e-mail: nibedita.mandal.iitkgp@gmail.com
pratima@maths.iitkgp.ernet.in

Abstract

An $L(2, 1)$ -coloring (or labeling) of a graph G is a vertex coloring $f : V(G) \rightarrow \mathbb{Z}^+ \cup \{0\}$ such that $|f(u) - f(v)| \geq 2$ for all edges uv of G , and $|f(u) - f(v)| \geq 1$ if $d(u, v) = 2$, where $d(u, v)$ is the distance between vertices u and v in G . The *span of an $L(2, 1)$ -coloring* is the maximum color (or label) assigned by it. The *span of a graph G* is the smallest integer λ such that there exists an $L(2, 1)$ -coloring of G with span λ . An $L(2, 1)$ -coloring of a graph with span equal to the span of the graph is called a *span coloring*. For an $L(2, 1)$ -coloring f of a graph G with span k , an integer h is a *hole* in f if $h \in (0, k)$ and there is no vertex v in G such that $f(v) = h$. A *no-hole coloring* is an $L(2, 1)$ -coloring with no hole in it. An $L(2, 1)$ -coloring is *irreducible* if color of none of the vertices in the graph can be decreased to yield another $L(2, 1)$ -coloring of the same graph. A graph G is *inh-colorable* if there exists an irreducible no-hole coloring of G . Most of the results obtained in this paper are answers to some problems asked by Laskar *et al.* [5]. These problems are mainly about relationship between the span and maximum no-hole span of a graph, lower inh-span and upper inh-span of a graph, and the maximum number of holes and minimum number of holes in a span coloring of a graph. We also give some sufficient conditions for a tree and an unicyclic graph to have inh-span $\Delta + 1$.

Keywords: $L(2, 1)$ -coloring, span of a graph, no-hole coloring, irreducible coloring, unicyclic graph.

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