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A MAXIMUM RESONANT SET OF POLYOMINO GRAPHS [1]

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Abstract

A polyomino graph P is a connected finite subgraph of the infinite plane grid such that each finite face is surrounded by a regular square of side length one and each edge belongs to at least one square. A dimer covering of P corresponds to a perfect matching. Different dimer coverings can interact via an alternating cycle (or square) with respect to them. A set of disjoint squares of P is a resonant set if P has a perfect matching M so that each one of those squares is M-alternating. In this paper, we show that if K is a maximum resonant set of P, then $P - K$ has a unique perfect matching. We further prove that the maximum forcing number of a polyomino graph is equal to the cardinality of a maximum resonant set. This confirms a conjecture of Xu *et al.* [26]. We also show that if K is a maximal alternating set of P, then $P - K$ has a unique perfect matching.

Keywords: polyomino graph, dimer problem, perfect matching, resonant set, forcing number, alternating set.

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