Discussiones Mathematicae
Graph Theory 36 (2016) 355–362
doi:10.7151/dmgt.1856

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DMGT Page

HARDNESS RESULTS FOR TOTAL RAINBOW CONNECTION OF GRAPHS

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Abstract

A total-colored path is *total rainbow* if both its edges and internal vertices have distinct colors. The *total rainbow connection number* of a connected graph G, denoted by $trc(G)$, is the smallest number of colors that are needed in a total-coloring of G in order to make G *total rainbow connected*, that is, any two vertices of G are connected by a total rainbow path. In this paper, we study the computational complexity of total rainbow connection of graphs. We show that deciding whether a given total-coloring of a graph G makes it total rainbow connected is NP-Complete. We also prove that given a graph G, deciding whether $trc(G) = 3$ is NP-Complete.

Keywords: total rainbow connection, computational complexity.

2010 Mathematics Subject Classification: 05C15, 05C40, 68Q25, 68R10.

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doi:10.1007/s00453-012-9689-4

Received 10 March 2015
Revised 7 July 2015
Accepted 7 July 2015