

HARDNESS RESULTS FOR TOTAL RAINBOW CONNECTION OF GRAPHS

LILY CHEN

*School of Mathematics Science
Huaqiao University
Quanzhou 362021, China*

e-mail: lily60612@126.com

BOFENG HUO

*Department of Mathematics
Qinghai Normal University
Xining, 810008, China*

e-mail: hbf@qhnu.edu.cn

AND

YINGBIN MA

*College of Mathematics and Information Science
Henan Normal University
Xinxiang 453007, China*

e-mail: mayingbincw@htu.cn

Abstract

A total-colored path is *total rainbow* if both its edges and internal vertices have distinct colors. The *total rainbow connection number* of a connected graph G , denoted by $trc(G)$, is the smallest number of colors that are needed in a total-coloring of G in order to make G *total rainbow connected*, that is, any two vertices of G are connected by a total rainbow path. In this paper, we study the computational complexity of total rainbow connection of graphs. We show that deciding whether a given total-coloring of a graph G makes it total rainbow connected is NP-Complete. We also prove that given a graph G , deciding whether $trc(G) = 3$ is NP-Complete.

Keywords: total rainbow connection, computational complexity.

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