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BOUNDS ON THE NUMBER OF EDGES OF EDGE-MINIMAL, EDGE-MAXIMAL AND l-HYPERTREES

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Abstract

In their paper, *Bounds on the number of edges in hypertrees*, G.Y. Katona and P.G.N. Szabó introduced a new, natural definition of hypertrees in k-uniform hypergraphs and gave lower and upper bounds on the number of edges. They also defined edge-minimal, edge-maximal and l-hypertrees and proved an upper bound on the edge number of l-hypertrees.

In the present paper, we verify the asymptotic sharpness of the $\binom{n}{k-1}$ upper bound on the number of edges of k-uniform hypertrees given in the above mentioned paper. We also make an improvement on the upper bound of the edge number of 2-hypertrees and give a general extension construction with its consequences.

We give lower and upper bounds on the maximal number of edges of k-uniform edge-minimal hypertrees and a lower bound on the number of edges of k-uniform edge-maximal hypertrees. In the former case, the sharp upper bound is conjectured to be asymptotically $\frac{1}{k-1}\binom{n}{2}$.

Keywords: hypertree, chain in hypergraph, edge-minimal hypertree, edge-maximal hypertree, 2-hypertree, Steiner system.

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