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BOUNDS ON THE DISJUNCTIVE TOTAL DOMINATION NUMBER OF A TREE

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Abstract

Let G be a graph with no isolated vertex. In this paper, we study a parameter that is a relaxation of arguably the most important domination parameter, namely the total domination number, $\gamma_t(G)$. A set S of vertices in G is a disjunctive total dominating set of G if every vertex is adjacent to a vertex of S or has at least two vertices in S at distance 2 from it. The disjunctive total domination number, $\gamma_t^d(G)$, is the minimum cardinality of such a set. We observe that $\gamma_t^d(G) \leq \gamma_t(G)$. A leaf of G is a vertex of degree 1, while a support vertex of G is a vertex adjacent to a leaf. We show that if T is a tree of order n with ℓ leaves and s support vertices, then $2(n-\ell+3)/5 \leq \gamma_t^d(T) \leq (n+s-1)/2$ and we characterize the families of trees which attain these bounds. For every tree T, we show have $\gamma_t(T)/\gamma_t^d(T) < 2$ and this bound is asymptotically tight.

Keywords: total domination, disjunctive total domination, trees.

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