

LIGHT GRAPHS IN PLANAR GRAPHS OF LARGE GIRTH

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Abstract

A graph H is defined to be light in a graph family \mathcal{G} if there exist finite numbers $\varphi(H, \mathcal{G})$ and $w(H, \mathcal{G})$ such that each $G \in \mathcal{G}$ which contains H as a subgraph, also contains its isomorphic copy K with $\Delta_G(K) \leq \varphi(H, \mathcal{G})$ and $\sum_{x \in V(K)} \deg_G(x) \leq w(H, \mathcal{G})$. In this paper, we investigate light graphs in families of plane graphs of minimum degree 2 with prescribed girth and no adjacent 2-vertices, specifying several necessary conditions for their lightness and providing sharp bounds on φ and w for light $K_{1,3}$ and C_{10} .

Keywords: planar graph, girth, light graph.

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REFERENCES

- [1] K. Appel and W. Haken, Every Planar Map is Four-Colorable (Providence, RI, American Mathematical Society, 1989).
doi:10.1090/conm/098
- [2] O.V. Borodin, *Solution of problems of Kotzig and Grünbaum concerning the isolation of cycles in planar graphs*, *Mat. Zametki* **46(5)** (1989) 9–12.

- [3] D.W. Cranston and D.B. West, *A guide to the discharging method*, arXiv:1306.4434 [math.CO] (2013).
- [4] S. Jendroľ and M. Maceková, *Describing short paths in plane graphs of girth at least 5*, Discrete Math. **338** (2015) 149–158.
doi:10.1016/j.disc.2014.09.014
- [5] S. Jendroľ, M. Maceková and R. Soták, *Note on 3-paths in plane graphs of girth 4*, Discrete Math. **338** (2015) 1643–1648.
doi:10.1016/j.disc.2015.04.011
- [6] S. Jendroľ, M. Maceková, M. Montassier and R. Soták, *Unavoidable 3-paths in planar graphs of given girth*, manuscript.
- [7] S. Jendroľ and P.J. Owens, *On light graphs in 3-connected plane graphs without triangular or quadrangular faces*, Graphs Combin. **17** (2001) 659–680.
doi:10.1007/s003730170007
- [8] S. Jendroľ and H.-J. Voss, *Light subgraphs of graphs embedded in the plane — A survey*, Discrete Math. **313** (2013) 406–421.
doi:10.1016/j.disc.2012.11.007
- [9] A. Kotzig, *Contribution to the theory of Eulerian polyhedra*, Mat. Čas. SAV (Math. Slovaca) **5** (1955) 101–113.
- [10] H. Lebesgue, *Quelques conséquences simples de la formule d’Euler*, J. Math. Pures Appl. **19** (1940) 27–43.
- [11] N. Robertson, D.P. Sanders, P.D. Seymour and R. Thomas, *The four-colour theorem*, J. Combin. Theory Ser. B **70** (1997) 2–44.
doi:10.1006/jctb.1997.1750
- [12] D.B. West, *Introduction to Graph Theory* (Prentice Hall, 2001).

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