

## LIGHT GRAPHS IN PLANAR GRAPHS OF LARGE GIRTH

PETER HUDÁK, MÁRIA MACEKOVÁ

TOMÁŠ MADARAS

AND

PAVOL ŠIROCZKI

*Institute of Mathematics  
P.J. Šafárik University in Košice  
Jesenná 5, 04001 Košice, Slovakia*

**e-mail:** peter.hudak@student.upjs.sk  
maria.macekova@student.upjs.sk  
tomas.madaras@upjs.sk  
sirocki@gmail.com

### Abstract

A graph  $H$  is defined to be light in a graph family  $\mathcal{G}$  if there exist finite numbers  $\varphi(H, \mathcal{G})$  and  $w(H, \mathcal{G})$  such that each  $G \in \mathcal{G}$  which contains  $H$  as a subgraph, also contains its isomorphic copy  $K$  with  $\Delta_G(K) \leq \varphi(H, \mathcal{G})$  and  $\sum_{x \in V(K)} \deg_G(x) \leq w(H, \mathcal{G})$ . In this paper, we investigate light graphs in families of plane graphs of minimum degree 2 with prescribed girth and no adjacent 2-vertices, specifying several necessary conditions for their lightness and providing sharp bounds on  $\varphi$  and  $w$  for light  $K_{1,3}$  and  $C_{10}$ .

**Keywords:** planar graph, girth, light graph.

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