Discussiones Mathematicae Graph Theory 36 (2016) 141–151 doi:10.7151/dmgt.1845

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ON AN EXTREMAL PROBLEM IN THE CLASS OF BIPARTITE 1-PLANAR GRAPHS

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Abstract

A graph G = (V, E) is called 1-planar if it admits a drawing in the plane such that each edge is crossed at most once. In this paper, we study bipartite 1-planar graphs with prescribed numbers of vertices in partite sets. Bipartite 1-planar graphs are known to have at most 3n - 8 edges, where ndenotes the order of a graph. We show that maximal-size bipartite 1-planar graphs which are almost balanced have not significantly fewer edges than indicated by this upper bound, while the same is not true for unbalanced

¹ This work was partially supported by the Polish Ministry of Science and Higher Education.

 $^{^2 \}rm This$ work was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0892. This work was supported by the Slovak Research and Development Agency under the contract No. APVV-0482-11, by the grants VEGA 1/0529/15, VEGA 1/0908/15 and KEGA 040TUKE4/2014."

ones. We prove that the maximal possible size of bipartite 1-planar graphs whose one partite set is much smaller than the other one tends towards 2n rather than 3n. In particular, we prove that if the size of the smaller partite set is sublinear in n, then |E| = (2 + o(1))n, while the same is not true otherwise.

Keywords: 1-planar graph, bipartite graph, graph size.2010 Mathematics Subject Classification: 05C10, 05C42, 05C62.

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Received 28 February 2015 Revised 27 May 2015 Accepted 27 May 2015