

ON AN EXTREMAL PROBLEM IN THE CLASS OF BIPARTITE 1-PLANAR GRAPHS

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Abstract

A graph $G = (V, E)$ is called 1-planar if it admits a drawing in the plane such that each edge is crossed at most once. In this paper, we study bipartite 1-planar graphs with prescribed numbers of vertices in partite sets. Bipartite 1-planar graphs are known to have at most $3n - 8$ edges, where n denotes the order of a graph. We show that maximal-size bipartite 1-planar graphs which are almost balanced have not significantly fewer edges than indicated by this upper bound, while the same is not true for unbalanced

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ones. We prove that the maximal possible size of bipartite 1-planar graphs whose one partite set is much smaller than the other one tends towards $2n$ rather than $3n$. In particular, we prove that if the size of the smaller partite set is sublinear in n , then $|E| = (2 + o(1))n$, while the same is not true otherwise.

Keywords: 1-planar graph, bipartite graph, graph size.

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REFERENCES

- [1] F.J. Brandenburg, D. Eppstein, A. Gleißner, M.T. Goodrich, K. Hanauer and J. Reislhuber, *On the density of maximal 1-planar graphs*, Graph Drawing, Lecture Notes Comput. Sci. **7704** (2013) 327–338.
doi:10.1007/978-3-642-36763-2_29
- [2] J. Czap and D. Hudák, *On drawings and decompositions of 1-planar graphs*, Electron. J. Combin. **20** (2013) P54.
- [3] J. Czap and D. Hudák, *1-planarity of complete multipartite graphs*, Discrete Appl. Math. **160** (2012) 505–512.
doi:10.1016/j.dam.2011.11.014
- [4] R. Diestel, Graph Theory (Springer, New York, 2010).
- [5] I. Fabrici and T. Madaras, *The structure of 1-planar graphs*, Discrete Math. **307** (2007) 854–865.
doi:10.1016/j.disc.2005.11.056
- [6] D.V. Karpov, *An upper bound on the number of edges in an almost planar bipartite graph*, J. Math. Sci. **196** (2014) 737–746.
doi:10.1007/s10958-014-1690-9
- [7] J. Pach and G. Tóth, *Graphs drawn with few crossings per edge*, Combinatorica **17** (1997) 427–439.
doi:10.1007/BF01215922
- [8] H. Bodendiek, R. Schumacher and K. Wagner, *Über 1-optimale Graphen*, Math. Nachr. **117** (1984) 323–339.
doi:10.1002/mana.3211170125
- [9] É. Sopena, personal communication, Seventh Cracow Conference in Graph Theory, Rytro (2014).

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