

VERTICES CONTAINED IN ALL OR IN NO MINIMUM SEMITOTAL DOMINATING SET OF A TREE

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Abstract

Let G be a graph with no isolated vertex. In this paper, we study a parameter that is squeezed between arguably the two most important domination parameters; namely, the domination number, $\gamma(G)$, and the total domination number, $\gamma_t(G)$. A set S of vertices in a graph G is a semitotal dominating set of G if it is a dominating set of G and every vertex in S is within distance 2 of another vertex of S. The semitotal domination number, $\gamma_{t2}(G)$, is the minimum cardinality of a semitotal dominating set of G. We observe that $\gamma(G) \leq \gamma_{t2}(G) \leq \gamma_t(G)$. We characterize the set of vertices that are contained in all, or in no minimum semitotal dominating set of a tree.

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Appendix

We now present an example to illustrate Theorem ??. Applying our pruning process discussed in Section ?? to the rooted tree T with root v illustrated in Figure 1(a), we proceed as follows.

• The branch vertices b_3 and b_4 are both at maximum distance 3 from v in T. We select b_3 , where $|L^3(b_3)| = 1$. Thus, b_3 is a type-(T.1) branch vertex and we delete $D(b_3)$ and attach a path of length 3 to b_3 .

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(b)) is the vertex b_4 . Since $|L^1(b_4)| > 2$ and every leaf-descendant of b_4 belongs to $L^1(b_4)$, the vertex b_4 is therefore a type-(T.3) branch vertex and we delete $D(b_4)$ and attach a path of length 1 to b_4 .

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(c)) is the vertex b_2 . Since $|L^4(b_2)| = 1$ and $L^1(b_2) = L^3(b_2) = \emptyset$, the vertex b_2 is a type-(T.4) branch vertex and we delete $D(b_2)$ and attach a path of length 4 to b_2 .

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(d)) is the vertex b_1 . Since $|L^3(b_1)| = 1$, the vertex b_1 is a type-(T.1) branch



Figure 1. The pruning of a tree rooted at v.

vertex and we delete $D(b_1)$ and attach a path of length 3 to b_1 . The resulting pruned tree \overline{T}_v is illustrated in Figure 1(e). • Since $|\overline{L}^1(v)| = 1$ and $|\overline{L}^4(v)| = 1$, by Theorem ??, we deduce that $v \notin \mathcal{A}_{t2}(T) \cup$

 $\mathcal{N}_{t2}(T).$

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