Discussiones Mathematicae
Graph Theory 36 (2016) 71–93
doi:10.7151/dmgt.1844

VERTICES CONTAINED IN ALL OR IN NO MINIMUM SEMITOTAL DOMINATING SET OF A TREE

Michael A. Henning[1]

and

Alister J. Marcon[2]

Department of Pure and Applied Mathematics
University of Johannesburg
Auckland Park, 2006, South Africa

e-mail: mahenning@uj.ac.za
alister.marcon@gmail.com

Abstract

Let G be a graph with no isolated vertex. In this paper, we study a parameter that is squeezed between arguably the two most important domination parameters; namely, the domination number, $\gamma(G)$, and the total domination number, $\gamma_t(G)$. A set S of vertices in a graph G is a semitotal dominating set of G if it is a dominating set of G and every vertex in S is within distance 2 of another vertex of S. The semitotal domination number, $\gamma_{t2}(G)$, is the minimum cardinality of a semitotal dominating set of G. We observe that $\gamma(G) \leq \gamma_{t2}(G) \leq \gamma_t(G)$. We characterize the set of vertices that are contained in all, or in no minimum semitotal dominating set of a tree.

Keywords: domination, semitotal domination, trees.

2010 Mathematics Subject Classification: 05C69.

References

[1] M. Blidia, M. Chellali and S. Khelifi, *Vertices belonging to all or no minimum double dominating sets in trees*, AKCE Int. J. Graphs. Comb. **2** (2005) 1–9.

[1]Research supported in part by the South African National Research Foundation and the University of Johannesburg.

[2]Funded by the South African National Research Foundation.

[2] E.J. Cockayne, M.A. Henning and C.M. Mynhardt, *Vertices contained in all or in no minimum total dominating set of a tree*, Discrete Math. **260** (2003) 37–44. doi:10.1016/S0012-365X(02)00447-8

[3] W. Goddard, M.A. Henning and C.A. McPillan, *Semitotal domination in graphs*, Util. Math. **94** (2014) 67–81.

[4] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, Inc. New York, 1998).

[5] M.A. Henning, *Recent results on total domination in graphs: A survey*, Discrete Math. **309** (2009) 32–63. doi:10.1016/j.disc.2007.12.044

[6] M.A. Henning and A.J. Marcon, *On matching and semitotal domination in graphs*, Discrete Math. **324** (2014) 13–18. doi:10.1016/j.disc.2014.01.021

[7] M.A. Henning and A.J. Marcon, *Semitotal domination in graphs: Partition and algorithmic results*, Util. Math., to appear.

[8] M.A. Henning and M.D. Plummer, *Vertices contained in all or in no minimum paired-dominating set of a tree*, J. Comb. Optim. **10** (2005) 283–294. doi:10.1007/s10878-005-4107-3

[9] M.A. Henning and A. Yeo, Total domination in graphs (Springer Monographs in Mathematics, 2013).

[10] C.M. Mynhardt, *Vertices contained in every minimum dominating set of a tree*, J. Graph Theory **31** (1999) 163–177. doi:10.1002/(SICI)1097-0118(199907)31:3⟨163::AID-JGT2⟩3.0.CO;2-T

Appendix

We now present an example to illustrate Theorem **??**. Applying our pruning process discussed in Section **??** to the rooted tree T with root v illustrated in Figure 1(a), we proceed as follows.

• The branch vertices b_3 and b_4 are both at maximum distance 3 from v in T. We select b_3, where $|L^3(b_3)| = 1$. Thus, b_3 is a type-(T.1) branch vertex and we delete $D(b_3)$ and attach a path of length 3 to b_3.

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(b)) is the vertex b_4. Since $|L^1(b_4)| > 2$ and every leaf-descendant of b_4 belongs to $L^1(b_4)$, the vertex b_4 is therefore a type-(T.3) branch vertex and we delete $D(b_4)$ and attach a path of length 1 to b_4.

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(c)) is the vertex b_2. Since $|L^4(b_2)| = 1$ and $L^1(b_2) = L^3(b_2) = \emptyset$, the vertex b_2 is a type-(T.4) branch vertex and we delete $D(b_2)$ and attach a path of length 4 to b_2.

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(d)) is the vertex b_1. Since $|L^3(b_1)| = 1$, the vertex b_1 is a type-(T.1) branch

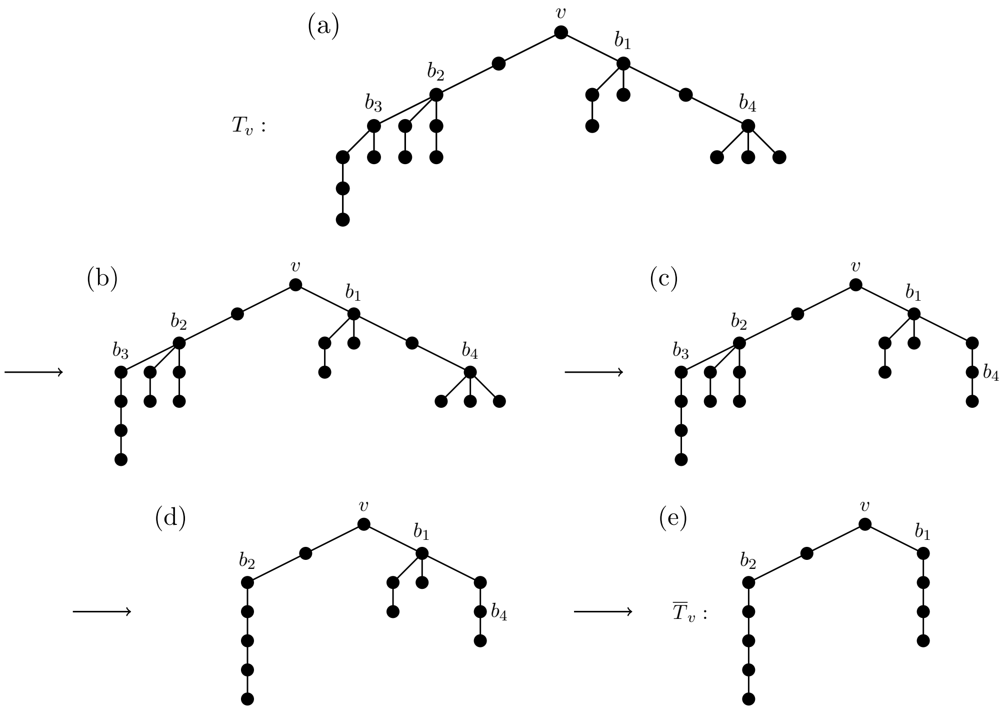

Figure 1.The pruning of a tree rooted at v.

vertex and we delete $D(b_1)$ and attach a path of length 3 to b_1. The resulting pruned tree $\overline{T}_v$ is illustrated in Figure 1(e).

• Since $|\overline{L}^1(v)| = 1$ and $|\overline{L}^4(v)| = 1$, by Theorem **??**, we deduce that $v \notin \mathcal{A}_{t2}(T) \cup \mathcal{N}_{t2}(T)$.

Received 18 December 2014
Revised 13 April 2015
Accepted 13 April 2015