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MAXIMUM EDGE-COLORINGS OF GRAPHS

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Abstract

An *r-maximum k-edge-coloring* of G is a k-edge-coloring of G having a property that for every vertex v of degree $d_G(v) = d$, $d \geq r$, the maximum color, that is present at vertex v, occurs at v exactly r times. The r-maximum index $\chi'_r(G)$ is defined to be the minimum number k of colors needed for an r-maximum k-edge-coloring of graph G. In this paper we show that $\chi'_r(G) \leq 3$ for any nontrivial connected graph G and $r = 1$ or 2. The bound 3 is tight. All graphs G with $\chi'_1(G) = i, i = 1, 2, 3$ are characterized. The precise value of the r-maximum index, $r \geq 1$, is determined for trees and complete graphs.

Keywords: edge-coloring, r-maximum k-edge-coloring, unique-maximum edge-coloring, weak-odd edge-coloring, weak-even edge-coloring.

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