

A FAN-TYPE HEAVY PAIR OF SUBGRAPHS FOR PANCYCLICITY OF 2-CONNECTED GRAPHS

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Abstract

Let G be a graph on n vertices and let H be a given graph. We say that G is pancyclic, if it contains cycles of all lengths from 3 up to n , and that it is H - f_1 -heavy, if for every induced subgraph K of G isomorphic to H and every two vertices $u, v \in V(K)$, $d_K(u, v) = 2$ implies $\min\{d_G(u), d_G(v)\} \geq \frac{n+1}{2}$. In this paper we prove that every 2-connected $\{K_{1,3}, P_5\}$ - f_1 -heavy graph is pancyclic. This result completes the answer to the problem of finding f_1 -heavy pairs of subgraphs implying pancyclicity of 2-connected graphs.

Keywords: cycle, Fan-type heavy subgraph, Hamilton cycle, pancyclicity.

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