

DOMINATION GAME CRITICAL GRAPHS

CSILLA BUJTÁS

Department of Computer Science and Systems Technology

University of Pannonia, Veszprém, Hungary

e-mail: bujtas@dcs.uni-pannon.hu

SANDI KLAVŽAR

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia

Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia

e-mail: sandi.klavzar@fmf.uni-lj.si

AND

GAŠPER KOŠMRLJ

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

e-mail: gasper.kosmrlj@student.fmf.uni-lj.si

Abstract

The domination game is played on a graph G by two players who alternately take turns by choosing a vertex such that in each turn at least one previously undominated vertex is dominated. The game is over when each vertex becomes dominated. One of the players, namely Dominator, wants to finish the game as soon as possible, while the other one wants to delay the end. The number of turns when Dominator starts the game on G and both players play optimally is the graph invariant $\gamma_g(G)$, named the game domination number. Here we study the γ_g -critical graphs which are critical with respect to vertex predomination. Besides proving some general properties, we characterize γ_g -critical graphs with $\gamma_g = 2$ and with $\gamma_g = 3$, moreover for each n we identify the (infinite) class of all γ_g -critical ones among the n th powers C_N^n of cycles. Along the way we determine $\gamma_g(C_N^n)$ for all n and N . Results of a computer search for γ_g -critical trees are presented and several problems and research directions are also listed.

Keywords: domination number, domination game, domination game critical graphs, powers of cycles, trees.

2010 Mathematics Subject Classification: 05C69, 05C57.

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Received 12 December 2014

Revised 19 March 2015

Accepted 19 March 2015