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MAXIMUM INDEPENDENT SETS IN DIRECT PRODUCTS OF CYCLES OR TREES WITH ARBITRARY GRAPHS

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Abstract

The direct product of graphs $G = (V(G), E(G))$ and $H = (V(H), E(H))$ is the graph, denoted as $G \times H$, with vertex set $V(G \times H) = V(G) \times V(H)$, where vertices (x_1, y_1) and (x_2, y_2) are adjacent in $G \times H$ if $x_1x_2 \in E(G)$ and $y_1y_2 \in E(H)$. Let n be odd and m even. We prove that every maximum independent set in $P_n \times G$, respectively $C_m \times G$, is of the form $(A \times C) \cup (B \times D)$, where C and D are nonadjacent in G, and $A \cup B$ is the bipartition of P_n respectively C_m. We also give a characterization of maximum independent subsets of $P_n \times G$ for every even n and discuss the structure of maximum independent sets in $T \times G$ where T is a tree.

Keywords: direct product, independent set.

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