Discussiones Mathematicae Graph Theory 35 (2015) 773–780 doi:10.7151/dmgt.1835

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UNIFIED SPECTRAL BOUNDS ON THE CHROMATIC NUMBER

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Abstract

One of the best known results in spectral graph theory is the following lower bound on the chromatic number due to Alan Hoffman, where μ_1 and μ_n are respectively the maximum and minimum eigenvalues of the adjacency matrix: $\chi \ge 1 + \mu_1 / - \mu_n$. We recently generalised this bound to include all eigenvalues of the adjacency matrix.

In this paper, we further generalize these results to include all eigenvalues of the adjacency, Laplacian and signless Laplacian matrices. The various known bounds are also unified by considering the normalized adjacency matrix, and examples are cited for which the new bounds outperform known bounds.

Keywords: chromatic number, majorization.2010 Mathematics Subject Classification: 97K30, 97H60, 05C50.

References

- R. Bhatia, Matrix Analysis (Graduate Text in Mathematics, 169, Springer Verlag, New York, 1997). doi:10.1007/978-1-4612-0653-8
- [2] F.R.K. Chung, Spectral Graph Theory (CBMS Number 92, 1997).

- [3] A.J. Hoffman, On eigenvalues and colourings of graphs, in: Graph Theory and its Applications, Academic Press, New York (1970) 79–91.
- [4] L. Yu. Kolotilina, Inequalities for the extreme eigenvalues of block-partitioned Hermitian matrices with applications to spectral graph theory, J. Math. Sci. 176 (2011) 44–56 (translation of the paper originally published in Russian in Zapiski Nauchnykh Seminarov POMI 382 (2010) 82–103).
- [5] L.S. de Lima, C.S. Oliveira, N.M.M. de Abreu and V. Nikiforov, *The smallest eigenvalue of the signless Laplacian*, Linear Algebra Appl. **435** (2011) 2570–2584. doi:10.1016/j.laa.2011.03.059
- V. Nikiforov, Chromatic number and spectral radius, Linear Algebra Appl. 426 (2007) 810–814. doi:10.1016/j.laa.2007.06.005
- [7] P. Wocjan and C. Elphick, New spectral bounds on the chromatic number encompassing all eigenvalues of the adjacency matrix, Electron. J. Combin. 20(3) (2013) P39.

Received 29 October 2014 Revised 10 March 2015 Accepted 10 March 2015