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UNIFIED SPECTRAL BOUNDS ON THE CHROMATIC NUMBER

Clive Elphick

Bretherton, UK

e-mail: clive.elphick@gmail.com

and

Pawel Wocjan

Department of Electrical Engineering and Computer Science
University of Central Florida
Orlando, FL 32816, USA

e-mail: wocjan@eecs.ucf.edu

Abstract

One of the best known results in spectral graph theory is the following lower bound on the chromatic number due to Alan Hoffman, where μ_1 and μ_n are respectively the maximum and minimum eigenvalues of the adjacency matrix: $\chi \geq 1 + \mu_1/-\mu_n$. We recently generalised this bound to include all eigenvalues of the adjacency matrix.

In this paper, we further generalize these results to include all eigenvalues of the adjacency, Laplacian and signless Laplacian matrices. The various known bounds are also unified by considering the normalized adjacency matrix, and examples are cited for which the new bounds outperform known bounds.

Keywords: chromatic number, majorization.

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