Discussiones Mathematicae
Graph Theory 35 (2015) 733–754
doi:10.7151/dmgt.1834

RAINBOW TETRAHEDRA IN CAYLEY GRAPHS

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Abstract

Let Γ_n be the complete undirected Cayley graph of the odd cyclic group $\mathbf{Z}_n$. Connected graphs whose vertices are rainbow tetrahedra in Γ_n are studied, with any two such vertices adjacent if and only if they share (as tetrahedra) precisely two distinct triangles. This yields graphs G of largest degree 6, asymptotic diameter $|V(G)|^{1/3}$ and almost all vertices with degree: (a) 6 in G; (b) 4 in exactly six connected subgraphs of the $(3,6,3,6)$-semi-regular tessellation; and (c) 3 in exactly four connected subgraphs of the $\{6,3\}$-regular hexagonal tessellation. These vertices have as closed neighborhoods the union (in a fixed way) of closed neighborhoods in the ten respective resulting tessellations.

Keywords: rainbow triangles, rainbow tetrahedra, Cayley graphs.

2010 Mathematics Subject Classification: 05C15, 05C75, 05C62.

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Received 6 November 2014
Revised 27 February 2015
Accepted 27 February 2015