

DECOMPOSITION OF COMPLETE BIPARTITE  
MULTIGRAPHS INTO PATHS  
AND CYCLES HAVING  $k$  EDGES

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**Abstract**

We give necessary and sufficient conditions for the decomposition of complete bipartite multigraph  $K_{m,n}(\lambda)$  into paths and cycles having  $k$  edges. In particular, we show that such decomposition exists in  $K_{m,n}(\lambda)$ , when  $\lambda \equiv 0 \pmod{2}$ ,  $m, n \geq \frac{k}{2}$ ,  $m + n > k$ , and  $k(p + q) = 2mn$  for  $k \equiv 0 \pmod{2}$  and also when  $\lambda \geq 3$ ,  $\lambda m \equiv \lambda n \equiv 0 \pmod{2}$ ,  $k(p + q) = \lambda mn$ ,  $m, n \geq k$ , (resp.,  $m, n \geq 3k/2$ ) for  $k \equiv 0 \pmod{4}$  (respectively, for  $k \equiv 2 \pmod{4}$ ). In fact, the necessary conditions given above are also sufficient when  $\lambda = 2$ .

**Keywords:** path, cycle, graph decomposition, multigraph.

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