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DECOMPOSITION OF COMPLETE BIPARTITE MULTIGRAPHS INTO PATHS AND CYCLES HAVING k EDGES

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Abstract

We give necessary and sufficient conditions for the decomposition of complete bipartite multigraph $K_{m,n}(\lambda)$ into paths and cycles having k edges. In particular, we show that such decomposition exists in $K_{m,n}(\lambda)$, when $\lambda \equiv 0 \pmod 2$, $m, n \geq \frac{k}{2}$, $m + n > k$, and $k(p + q) = 2mn$ for $k \equiv 0 \pmod 2$ and also when $\lambda \geq 3$, $\lambda m \equiv \lambda n \equiv 0 \pmod 2$, $k(p + q) = \lambda mn$, $m, n \geq k$, (resp., $m, n \geq 3k/2$) for $k \equiv 0 \pmod 4$ (respectively, for $k \equiv 2 \pmod 4$). In fact, the necessary conditions given above are also sufficient when $\lambda = 2$.

Keywords: path, cycle, graph decomposition, multigraph.

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