

CHROMATIC SUMS FOR COLORINGS AVOIDING MONOCHROMATIC SUBGRAPHS¹

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Abstract

Given graphs G and H , a vertex coloring $c : V(G) \rightarrow \mathbb{N}$ is an H -free coloring of G if no color class contains a subgraph isomorphic to H . The H -free chromatic number of G , $\chi(H, G)$, is the minimum number of colors in an H -free coloring of G . The H -free chromatic sum of G , $\Sigma(H, G)$, is the minimum value achieved by summing the vertex colors of each H -free coloring of G . We provide a general bound for $\Sigma(H, G)$, discuss the computational complexity of finding this parameter for different choices of H , and prove an exact formulas for some graphs G . For every integer k and for every graph H , we construct families of graphs, G_k with the property that k more colors than $\chi(H, G)$ are required to realize $\Sigma(H, G)$ for H -free colorings. More complexity results and constructions of graphs requiring extra colors are given for planar and outerplanar graphs.

Keywords: coloring, sum of colors, forbidden subgraphs.

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