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CHROMATIC SUMS FOR COLORINGS AVOIDING MONOCHROMATIC SUBGRAPHS[1]

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Abstract

Given graphs G and H, a vertex coloring $c : V(G) \to \mathbb{N}$ is an H-free coloring of G if no color class contains a subgraph isomorphic to H. The H-free chromatic number of G, $\chi(H, G)$, is the minimum number of colors in an H-free coloring of G. The H-free chromatic sum of G, $\Sigma(H, G)$, is the minimum value achieved by summing the vertex colors of each H-free coloring of G. We provide a general bound for $\Sigma(H, G)$, discuss the computational complexity of finding this parameter for different choices of H, and prove an exact formulas for some graphs G. For every integer k and for every graph H, we construct families of graphs, G_k with the property that k more colors than $\chi(H, G)$ are required to realize $\Sigma(H, G)$ for H-free colorings. More complexity results and constructions of graphs requiring extra colors are given for planar and outerplanar graphs.

Keywords: coloring, sum of colors, forbidden subgraphs.

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