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# THE SATURATION NUMBER FOR THE LENGTH OF DEGREE MONOTONE PATHS

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**Abstract**

A degree monotone path in a graph $G$ is a path $P$ such that the sequence of degrees of the vertices in the order in which they appear on $P$ is monotonic. The length (number of vertices) of the longest degree monotone path in $G$ is denoted by $mp(G)$. This parameter, inspired by the well-known Erdős-Szekeres theorem, has been studied by the authors in two earlier papers. Here we consider a saturation problem for the parameter $mp(G)$. We call $G$ saturated if, for every edge $e$ added to $G$, $mp(G+e) > mp(G)$, and we define $h(n,k)$ to be the least possible number of edges in a saturated graph $G$ on $n$ vertices with $mp(G) < k$, while $mp(G+e) \geq k$ for every new edge $e$.

We obtain linear lower and upper bounds for $h(n,k)$, we determine exactly the values of $h(n,k)$ for $k = 3$ and 4, and we present constructions of saturated graphs.

**Keywords:** paths, degrees, saturation.

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