# DECOMPOSABILITY OF ABSTRACT AND PATH-INDUCED CONVEXITIES IN HYPERGRAPHS 

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#### Abstract

An abstract convexity space on a connected hypergraph $H$ with vertex set $V(H)$ is a family $C$ of subsets of $V(H)$ (to be called the convex sets of $H)$ such that: (i) $C$ contains the empty set and $V(H)$, (ii) $C$ is closed under intersection, and (iii) every set in $C$ is connected in $H$. A convex set $X$ of $H$ is a minimal vertex convex separator of $H$ if there exist two vertices of $H$ that are separated by $X$ and are not separated by any convex set that is a proper subset of $X$. A nonempty subset $X$ of $V(H)$ is a cluster of $H$ if in $H$ every two vertices in $X$ are not separated by any convex set. The cluster hypergraph of $H$ is the hypergraph with vertex set $V(H)$ whose edges are the maximal clusters of $H$. A convexity space on $H$ is called decomposable if it satisfies the following three properties:


(C1) the cluster hypergraph of $H$ is acyclic,
(C2) every edge of the cluster hypergraph of $H$ is convex,
(C3) for every nonempty proper subset $X$ of $V(H)$, a vertex $v$ does not belong to the convex hull of $X$ if and only if $v$ is separated from $X$ in $H$ by a convex cluster.

It is known that the monophonic convexity (i.e., the convexity induced by the set of chordless paths) on a connected hypergraph is decomposable.

In this paper we first provide two characterizations of decomposable convexities and then, after introducing the notion of a hereditary path family in a connected hypergraph $H$, we show that the convexity space on $H$ induced
by any hereditary path family containing all chordless paths (such as the families of simple paths and of all paths) is decomposable.
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